Modelling the impacts of uncertainty and attitudes towards risk on production decisions in arable farming

by

Professor Dr. Ernst Berg

University of Bonn
Department of Farm Management
Meckenheimer Allee 174
D-53115 Bonn, Germany

Tel: +49 228 73-2891
Fax: +49 228 73-2758

e-mail: E.Berg@uni-bonn.de

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Abstract

This paper investigates the impacts of risk aversion on the choice of input levels and the optimal production program of arable farms. The analysis is conducted in an expected value-variance framework. A stochastic production function approach is used to analyse the risk associated with different levels of production intensity and to investigate the influence of risk aversion on optimal input use. In a second step the production functions are incorporated in a whole-farm stochastic programming approach that simultaneously optimises crop mix and production intensity. A case study is used to illustrate how uncertainty of yields and prices influences production decisions at different degrees of risk aversion.

Keywords: expected value-variance analysis, portfolio selection, risk management, stochastic programming

1 Introduction

Recent and presumable future developments tend to increase the risk associated with farming activities. Globalisation and liberalisation of trade combined with declining commodity price support result in an increase of market risks. Besides this, more stringent regulations with respect to the application of agro chemicals cause an increase of yield variability. In animal production, the achieved degree in the division of labour has dramatically aggravated the consequences of contagious disease outbreaks. This list could easily be extended. In summary it illustrates the increasing importance of analyses, which address issues of risk and uncertainty.

Farmers have a wide variety of possibilities to influence the risk exposure of their operations. Among them are the selection of variable input levels and the choice of the production program. In our paper we address these two issues including the interdependencies between them. A stochastic production function approach is used to analyse the risk associated with various levels of production intensity and to investigate the influence of risk aversion on the optimal input use in an expected value-variance framework. In the second part of the paper this approach is incorporated in a whole farm model that simultaneously optimises a portfolio of production activities along with the levels of production intensity for all production processes.

2 Theoretical background

The most general approach for comparing risky choices is by means of expected utility. This requires that all possible outcomes of the risky prospect be translated into utility measures to compute the expected utility. Faced with a choice amongst a set of risky prospects, the expected utility hypothesis states that the prospect with the highest utility is preferred. The expected utility can be retranslated into a monetary measure, i.e. the certainty equivalent (CE), through the inverse of the utility function. The certainty equivalent represents the certain amount of money, which a decision maker with a given utility function would rate as equivalent to the uncertain outcome of the risky prospect (cf. Robison and Barry, 1987, p. 23ff). Ranking prospects by CE is equivalent to ranking them by expected utility.

By definition the certainty equivalent $CE$ equals the expected return $E(y)$ minus the risk premium $\pi$, i.e. $CE = E(y) - \pi$. For the latter Pratt has derived the approximate relationship $p = \frac{1}{2} R[E(y)] V(y)$, where $R[E(y)]$ indicates the decision maker’s absolute risk aversion measured at the expected value $E(y)$ and $V(y)$ denotes the variance (cf. Robison and Barry, 1987, p. 34). Thus the certainty equivalent can be expressed as
The absolute risk aversion function defined as $R(y) = -U''(y)/U'(y)$ and can represent constant (CARA), decreasing (DARA) or increasing absolute risk aversion (IARA) with increasing outcome. For CARA equation (1) becomes

$$CE = E(y) - \frac{1}{2} R[E(y)] V(y)$$

where $\lambda$ denotes the constant measure of absolute risk aversion. The most usual assumption is that decision makers are said to display decreasing absolute risk aversion (Hardaker, 2000, p. 8). For the purpose of our study this can be considered by parameterisation of $\lambda$.

The outcome variable $y$ in equation (2) is normally expressed in terms of wealth. If we assume that final wealth is composed of a certain initial wealth plus the period income, which in turn is affected by the choice variables, then for CARA the analysis can equally be based on income (Robison and Barry, 1997, p. 126). In the remainder of this study we therefore measure $y$ in terms of net farm income.

The conditions under which the expected value-variance (EV) approach yields results consistent with the more general expected utility (EU) model have been worked out by several authors (cf. Meyer, 1987; Robison and Barry, 1987; Robison and Hanson, 1997). Even if these conditions are not completely met, the EV approach has proven its usefulness because of its deductive strength and the straightforward applicability in optimisation models. These are the main reasons for choosing the EV approach in the context of this study.

3 Uncertainty and optimal input use

We begin our study by assessing the problem of optimal input use under yield and price uncertainty. Later on the analysis is extended as to include decisions regarding the choice of the production program.

3.1 Production function approach

The analysis of optimal input and output levels in crop production is often based upon yield response functions reflecting the law of diminishing returns. While many empirical studies employ production functions of that nature (e.g. Schulte 1984, Weinschenck and Gebhardt 1985, Krayl et al. 1990) this approach seems hardly consistent with the fact that the composition of yield is chemically fixed and the components must be supplied as growth factors. Latter rather indicates the existence of linear input output relationships. The assumption of linearity is in accordance with Liebig’s well-known principle of the minimum factor that imposes an upper limit on the achievable output level. Some authors therefore suggest to use linear response functions, which they have found suitable for representing crop response to nutrients in homogeneous settings of soil and climate conditions (cf. Grimm et al. 1987, Paris and Knapp 1989, Bäumer 1994, Wagner 1995).

The apparent contradiction between Liebig’s minimum principle and the law of diminishing returns can be resolved if one considers the variability of growing conditions and the imperfect knowledge about them. Kuhlmann has shown this for the case of genetic variability within a plant population (Kuhlmann, 1992; Kuhlmann and Frick, 1995). Here the focus is on the variability and the stochastic nature of climate conditions (cf. Berg, 1997).

Liebig’s principle can be represented by a linear production function (Leontief production function) of the form

$$y = \text{Min} \left( \frac{x_1}{a_1}, \frac{x_2}{a_2}, \ldots, \frac{x_n}{a_n} \right)$$
where \( y \) denotes the output, \( x_i (i=1,2,...,n) \) is the level of the i-th input and the \( a_i (i=1,2,...,n) \) are constant production coefficients, representing the necessary input of the respective factor per unit of output. This formulation states that the total production amounts to the smallest ratio inside the brackets and that the subscript associated with this ratio marks the minimum factor, which limits the output. The \( x_i \) include controllable as well as non-controllable inputs.

Assuming that nitrogen is the relevant controllable input, the above relationship can be represented as given in Figure 1. The yield level increases linearly with the available nitrogen, where the total amount of nutrient is composed of the fertiliser nitrogen plus the nitrogen supply from the soil. Latter marks the intersection of the production function with the ordinate. The total production is constrained by the availability of other growth factors (e.g. temperature, solar radiation, water, pest damage, etc.). One of those represents the minimum factor, if sufficient nutrients are supplied.

In this setting the production function denotes the situation of a particular year with a given weather pattern. At the point in time, when decisions have to be made, this pattern is unknown so the maximum achievable yield as well as the nitrogen mineralisation from the soil must be regarded as uncertain parameters. Thus the system is stochastic in nature and can be formulated as follows:

\[
y = a^{-1}(x + s) \quad \text{for} \ y \leq y_{\text{max}} \quad \text{and} \quad y = y_{\text{max}} \quad \text{otherwise}
\]

with

\[
y_{\text{max}} = N\{\bar{y}_{\text{max}}, \sigma_{y_{\text{max}}} \} \quad \text{and} \quad s = N\{\bar{s}, \sigma_s \}
\]

The parameters \( y_{\text{max}} \) (maximum yield) and \( s \) (nitrogen supply from the soil) are assumed to be uncorrelated random variables that are normally distributed with the given means and standard deviations.\(^1\)

Based on these assumptions stochastic simulation can be used to study the yield response to varying nitrogen inputs. Production functions for different crops were simulated, using the parameters given in Table 1, which were derived from field records and from the literature. Figure 2 depicts the simulation results for wheat in terms of the average yield response and the variance of yield associated with different levels of fertiliser input. The markers represent

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\(^1\) In reality one would certainly expect these variables to be correlated, since they basically depend on the same climatic factors. We abstract from this for simplification reasons and because no information on the direction and magnitude of a possible correlation is available.
the results as obtained from the stochastic simulation model. Regression analysis is then used to fit quadratic functions to the simulation results. The estimated equations are also given in Figure 2. The yield response function exhibits diminishing marginal returns. Starting from a low input level, the variance of yields decreases with increasing levels of fertilisation. Thus, in a certain range nitrogen can be seen as a risk reducing input. The expected yield and variance functions will be used for the further analysis. Table 2 denotes the estimated functions for different crops along with some additional information.

Table 1: Simulation parameters

<table>
<thead>
<tr>
<th>Crop</th>
<th>( \bar{y}_{\text{max}} ) dt/ha</th>
<th>( \sigma_{y_{\text{max}}} ) dt/ha</th>
<th>( a )</th>
<th>( \sigma_x = 50 ) kg N/ha</th>
</tr>
</thead>
<tbody>
<tr>
<td>winter wheat</td>
<td>85</td>
<td>15</td>
<td>2,50</td>
<td>75 kg N/ha</td>
</tr>
<tr>
<td>winter barley</td>
<td>81</td>
<td>15</td>
<td>2,22</td>
<td>50 kg N/ha</td>
</tr>
<tr>
<td>winter rye</td>
<td>77</td>
<td>14</td>
<td>2,04</td>
<td></td>
</tr>
<tr>
<td>spring barley</td>
<td>67</td>
<td>10</td>
<td>2,22</td>
<td></td>
</tr>
<tr>
<td>oats</td>
<td>69</td>
<td>11</td>
<td>2,27</td>
<td></td>
</tr>
<tr>
<td>oil seed rape</td>
<td>41</td>
<td>8</td>
<td>3,85</td>
<td></td>
</tr>
</tbody>
</table>

mobilised nitrogen from soil: \( \bar{x} = 75 \) kg N/ha \( \sigma_x = 50 \) kg N/ha

Figure 2: Mean and variance of yield as function of nitrogen input
Table 2: Estimated expected values and variances of yields as functions of nitrogen input

<table>
<thead>
<tr>
<th></th>
<th>winter wheat</th>
<th>winter barley</th>
<th>winter rye</th>
<th>spring barley</th>
<th>oats</th>
<th>oil seed rape</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>expected value functions</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>constant</td>
<td>·x^0</td>
<td>2.797E+01</td>
<td>3.591E+01</td>
<td>4.249E+01</td>
<td>3.869E+01</td>
<td>2.235E+01</td>
</tr>
<tr>
<td>linear coefficient</td>
<td>·x^1</td>
<td>4.717E-01</td>
<td>4.162E-01</td>
<td>3.394E-01</td>
<td>2.688E-01</td>
<td>3.005E-01</td>
</tr>
<tr>
<td><strong>variance functions</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>constant</td>
<td>·x^0</td>
<td>5.108E+02</td>
<td>5.376E+02</td>
<td>5.248E+02</td>
<td>4.246E+02</td>
<td>4.417E+02</td>
</tr>
<tr>
<td>linear coefficient</td>
<td>·x^1</td>
<td>-2.965E+00</td>
<td>-3.289E+00</td>
<td>-3.356E+00</td>
<td>-3.250E+00</td>
<td>-3.205E+00</td>
</tr>
<tr>
<td>quadratic coefficient</td>
<td>·x^2</td>
<td>7.598E-03</td>
<td>8.794E-03</td>
<td>8.827E-03</td>
<td>8.242E-03</td>
<td>8.154E-03</td>
</tr>
</tbody>
</table>

**maximum expected yield solution**
- nitrogen fertilizer kg N/ha: 238.8, 216.4, 205.6, 201.0, 204.4, 206.8
- expected value of yield dt/ha: 84.3, 80.9, 77.4, 67.5, 69.4, 41.2
- standard deviation of yield dt/ha: 15.4, 15.4, 14.4, 10.2, 11.3, 8.2

**minimum variance solution**
- nitrogen fertilizer kg N/ha: 195.1, 187.0, 190.1, 197.2, 196.5, 184.4
- expected value of yield dt/ha: 82.4, 80.1, 77.2, 67.5, 69.4, 40.9
- standard deviation of yield dt/ha: 14.9, 15.2, 14.3, 10.2, 11.3, 8.2

**expected values and standard deviations of yields at 50 kg N/ha fertilizer level**
- expected value of yield dt/ha: 49.1, 54.3, 57.4, 52.2, 51.9, 30.4
- standard deviation of yield dt/ha: 19.5, 19.9, 19.5, 16.8, 17.4, 10.4

results of 2000 random simulation experiments
3.2 Mean variance analysis

Following we analyse the impact of risk aversion on the choice of input levels through EV analysis. Let \( p \) be the output price, \( c_1 \) the proportional costs of input \( x \) and \( c_0 \) the costs that are independent from the input level, then the expected return \( E(y) \) becomes

\[
E(y) = p \ E[f(x)] - c_0 - c_1 x
\]

where \( f(x) \) denotes the yield response function and \( E[\cdot] \) indicates the expectation operator. Assuming the cost to be non-stochastic, the variance of returns is given by

\[
V(y) = p^2 V[f(x)]
\]

where \( V[f(x)] \) denotes the variance function. To find the optimal input level we maximise the certainty equivalent for a given degree of absolute risk aversion:

\[
\text{max } CE = E(y) - \frac{\lambda}{2} V(y)
\]

Substituting \( E(y) \) and \( V(y) \) by equations (5) and (6) yields:

\[
\text{max } CE = p \ E[f(x)] - c_0 - c_1 x - \frac{\lambda}{2} p^2 V[f(x)]
\]

The first order condition is

\[
p \frac{d}{dx} E[f(x)] - c_1 - \frac{\lambda}{2} p^2 \frac{d}{dx} V[f(x)] = 0
\]

or by re-arranging the terms:

\[
\frac{d}{dx} E[f(x)] = \frac{c_1}{p} + \frac{\lambda}{2} \frac{d}{dx} V[f(x)]
\]

Since the variance decreases at increasing levels of \( x \), the derivative of \( V[f(x)] \) is negative until its minimum is achieved. Thus, risk is decreasing with growing \( x \) up to the minimum variance input. The result is that uncertainty associated with the effectiveness of inputs induces risk averse decision makers to use these inputs at higher levels, as long as the minimum variance input falls above the one that maximises expected returns. This is consistent with the results that Robison and Barry (1987, 118pp) have derived for the risk associated with the quality of inputs.

So far the commodity price \( p \) has been treated as deterministic. Extending the analysis to include stochastic product prices results in the following equation for the expected return:

\[
E(y) = E(p) \ E[f(x)] - c_0 - c_1 x + \text{cov}[f(x), p]
\]

Here \( \text{cov}[f(x), p] \) represents the covariance between yields and prices. The variance can be approximated by the following expression (Anderson et al., 1977, p. 32ff):

\[
V(y) = E(p)^2 V[f(x)] + V(p) E[f(x)]^2 + 2E(p) E[f(x)] \text{cov}[f(x), p]
\]

According to the above equations a negative correlation between yields and prices decreases the expected value but also the variance of returns, relative to the case of stochastic independence. The maximisation problem of equation (8) can now be rewritten as:

\[
\text{max } CE = E(p) \ E[f(x)] - c_0 - c_1 x + \text{cov}[f(x), p]
\]

\[
- \frac{\lambda}{2} \left\{ E(p)^2 V[f(x)] + V(p) E[f(x)]^2 + 2E(p) E[f(x)] \text{cov}[f(x), p] \right\}
\]


On neglecting the covariance terms for simplification the first order condition becomes:

\[
E(p) \frac{d}{dx} E[f(x)] - c_1 - \frac{\lambda}{2} \left\{ E(p)^2 \frac{d}{dx} V[f(x)] + 2 V(p) E[f(x)] \frac{d}{dx} E[f(x)] \right\} = 0
\]

Re-arranging the terms finally yields:

\[
\frac{d}{dx} E[f(x)] = \frac{c_1}{E(p) - \lambda V(p) E[f(x)]} + \frac{\lambda}{2} \frac{E(p)^2}{E(p) - \lambda V(p)} \frac{d}{dx} V[f(x)]
\]  

(13)

In case of a non-stochastic price the variance \( V(p) \) equals 0 and equation (13) reduces to equation (9). Increasing market risk, i.e. increasing variance of prices, has a two-fold impact: On one hand the denominator of the first fraction declines, thereby increasing the ratio. This means that the optimal nitrogen input moves to a lower level, i.e. towards a higher marginal expected yield. Thus, for a risk averse decision maker increasing price uncertainty c.p. leads to lower production intensity. At the same time the denominator of the second fraction declines as well which results in an increased ratio. Since the first derivative of \( V[f(x)] \) is negative until the minimum variance input level is achieved, the second fraction has a negative sign. This effect moves the optimum towards higher input levels. The intensity increasing effect of yield uncertainty stated before is therefore amplified by the price uncertainty. Which of these reverse impacts finally prevails depends on the constellation of parameters. Growing risk aversion causes an increase of production intensity, if the numerator of the second fraction in equation (13) is greater than \( c_1 \) and vice versa. Thus, high input levels are forced by low intensity dependent cost figures and good possibilities for variance reduction, latter represented by the derivative of the variance function. In contrast to this, high marginal cost and considerable price uncertainty cause decreasing input levels with increasing risk aversion.

4 Optimal crop mix

The multi-commodity operations that are typical for Europe always have the possibility to influence their risk exposure by the choice of crop mix. Capturing these effects requires a whole-farm approach that optimises a portfolio of production activities while for each production process the optimal level of intensity is chosen simultaneously. Following we develop a stochastic programming model of that nature.

4.1 Stochastic programming model

The objective function of the optimisation model is to maximise the certainty equivalent of income:

\[
\max \ CE = E(Y) - \frac{\lambda}{2} V(Y)
\]  

(14)

In the above equation \( E(Y) \) represents the expected value and \( V(Y) \) the variance of net farm income. The expected value of net farm income or profit results from the sum of gross margins of the production activities \( E(GM_i) \) multiplied by their respective acreage \( v_i \) after deducting the fixed cost \( FC \):

\[
E(Y) = \sum_{i=1}^{n} E(GM_i) v_i - FC
\]  

(15)

On replacing the \( E(GM_i) \) by the right hand side of equation (10) the expected profit can be rewritten as
\[ E(Y) = \sum_{i=1}^{n} \left( E(p_i) E[f_i(x_i)] - c_{0i} - c_{1i}x_i + \text{cov}[f_i(x_i), p_i] \right) v_i - FC \]  

(16)

Since all costs are assumed to be deterministic, the variance of income equals the variance of total revenue and can be computed from the variances and covariances of the commodity returns:

\[ V(Y) = \sum_{i=1}^{n} V(GM_i) v_i^2 + 2 \sum_{i=1}^{n} \sum_{j=i+1}^{n} v_i v_j \text{cov}(GM_i, GM_j) \]  

(17)

The covariances of the gross margins are mainly determined by the correlations between the crop yields. Since various studies indicate that these correlations are mostly close to zero (e.g. Rasmussen, 1997) we omit the covariance terms in the above equation for simplification. Using equation (11) the variance of income can be expressed as

\[ V(Y) = \sum_{i=1}^{n} \left( E(p_i)^2 V[f_i(x_i)] + V(p_i) E[f_i(x_i)]^2 + 2 E(p_i) E[f_i(x_i)] \text{cov}[f_i(x_i), p_i] \right) v_i^2 \]  

(18)

The decision variables of this model are the planted acreages \( v_i \) of the crops on one hand and the intensity levels \( x_i \) on the other hand. Hedging with futures and options are not included in the analysis since these activities have not yet gained much importance in Europe. The optimisation is subject to the constraints

\[ b_j \geq \sum_{i=1}^{n} a_{ij} v_i \quad \text{and} \quad x_i \geq 0; v_i \geq 0 \]  

(19)

These indicate that the total requirements of the production activities must not exceed the respective resources \( b_j \) (land, labour, etc.). Furthermore, all decision variables must be non-negative.

In the above form the model incorporates a non-linear optimisation problem, which can only be resolved using non-linear (numerical) optimisation procedures. In our case Microsoft EXCEL was used along with the included optimisation package SOLVER.

### 4.2 Model results

The following case study shall illustrate the combined effects of different degrees of risk aversion on optimal levels of intensity and the choice of the production program. The model calculations refer to a German arable farm. The farm size is 150 ha. The production activities that can be chosen are those of Table 2 along with potatoes and the necessary land set aside to obtain the area payments according to the European agricultural policy. Price expectations as well as cost figures were derived from field records collected by the extension service. To account for price risk, a coefficient of variation of 15% was assumed for all cereals and oil seed rape, while for potatoes this figure was set to 60%. Furthermore potatoes exhibit a statistically significant negative correlation between price and yield (cf. Treskow, 1983), which was considered in the model by applying a coefficient of correlation of –0.8. For all other crops the assumption is that yields and prices are stochastically independent. With respect to further model assumptions it shall be mentioned that the shares of potatoes and oil seed rape are respectively restricted to a maximum of 25% and that of winter wheat to 40% of the total acreage via rotational constraints.

To illustrate the effect of an increasing risk aversion the absolute risk aversion parameter \( \gamma \) was incremented across the model runs. The resulting changes in crop mix are depicted
in Figure 3. Starting with a risk neutral decision maker (i.e. \( \lambda = 0 \)) we find a rather specialised production program, where potatoes and wheat are grown at their respective limits according to the rotational constraints and the rest of the acreage is devoted to spring barley and the mandatory land set aside. Risk aversion causes an immediate reduction of potatoes because of the high price uncertainty associated with this crop. Instead, winter barley and at higher levels of risk aversion oil seed rape, oats and rye enter the production program. This diversification process is followed by a de-concentration in the way that the shares of relatively low-risk crops like cereals or oil seed rape increase while the acreage of potatoes is continuously reduced. The case of extreme risk aversion is represented by the minimum variance solution, in which potatoes have almost disappeared from the production program.

The changes of production intensity are represented in Figure 3 by the amount of nitrogen fertiliser (measured in kg N) that is applied per ha of total acreage. In the course of diversification, the level of intensity starts with a slight increase, which is followed by a decrease of similar extent caused by the inclusion of relatively low input crops (oilseed rape, oats, rye) in the production program. Later on the growing risk aversion leads to higher levels of intensity for all crops, which is reflected by the ascending line in Figure 3.

Diversification reduces risk but also leads to a decrease of expected income. This context is depicted by the expected value-standard deviation (ESD) diagram given in Figure 4. The standard deviation was chosen instead of the variance because it has the same dimension as the expected value. The starting point at the upper right corner of the ESD-frontier marks the case of risk indifference that yields the maximum expected profit. Through the changes of crop mix and production intensity as discussed earlier, the variability of income can be reduced, but only at the expense of a decreasing average income. After all it is up to the decision maker to determine which trade-off between expected value and standard deviation is still acceptable.
5 Conclusions

If we accept the hypothesis that risk aversion rather than risk indifference is the standard attitude of farmers, then we can conclude from the model results that uncertainty of crop yields and prices significantly influences production decisions. This is true for decisions regarding the level of intensity as well as for the choice of the production program. With respect to the former we have shown that risk aversion causes higher input levels than expected if certainty or risk indifference is proposed. This effect is particularly pronounced if yield uncertainty is the only (or most important) source of risk. The influence of price uncertainty on input use is somewhat ambiguous. In absence of yield uncertainty increasing price risk would always reduce the level of intensity. However, when both sources of risk are present the direction of changes depends on the specific constellation of parameters. With respect to crop mix decisions increasing risk aversion always leads to a more diversified production program and vice versa.

These effects are also important for assessing the response of farmers to policy changes or changes of economic conditions. If risk aversion prevails, the assessment of producer reactions on the basis of expected values is likely to yield biased results. From our analysis we can for instance conclude that in the presence of risk aversion changes of production intensity which result from changes of commodity prices will most likely be smaller than commonly expected. Policy measures, which affect the stability of markets, cause adjustments of the production program beyond those that follow directly from the changes of relative prices. Last not least the risk exposure of farms is influenced by the direct payments of the European agricultural policy. Assessing these effects requires that uncertainty and risk attitudes be explicitly considered in model calculations.

References


