GTAP in GAMS, Version 6.2

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Abstract

The purpose of this paper is to describe a version of the GTAP model implemented in GAMS that is a literal implementation of the standard GEMPACK version. There are multiple other GAMS models that rely on the basic GTAP database structure, but none are exact replicas of the GTAP model itself. This paper relies on version 6.2 of the GTAP model released in 2003 and that remains the standard GTAP model, acknowledging that there are multiple variants. The code has been tested with release 8 of the GTAP database that has 57 commodities and 134 countries/regions.

*All correspondence should be sent to vandermd@purdue.edu. The paper benefitted from the excellent comments of my colleagues at the Center for Global Trade Analysis, in particular Iman Haqiqi who read the paper with considerable care.

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1 Introduction

This paper provides a full specification of the standard GTAP model, version 6.2a, in GAMS. It is intended to complement the GEMPACK version of the model as developed in Hertel [1997]. It differs from the GAMS model developed by Rutherford (Rutherford [2010]) in a number of ways.[1] First, it is intended to be a full blown translation of the TABLO code. The Rutherford version is a variant that captures most of the structural features of the standard GTAP model, but has, for example, a simplified household demand structure. Second, the model is written and coded in the Derviş, de Melo and Robinson (Derviş et al. [1982]) tradition and departs significantly from the nomenclature used in the GEMPACK version of the model and that used by Rutherford. Though it is intended to replicate the GEMPACK version of the GTAP model, it also contains some extensions. Among these are:

1. The model differentiates between production activities and commodities, though can default to the standard practice of assuming a one-to-one mapping.

2. The production nesting allows for an aggregate intermediate demand bundle that is then decomposed into intermediate demand for goods and services.

3. Investment expenditures are specifically identified and not part of firms' activities.

4. Investment and government expenditures are allowed to have a generic CES expenditure. The former is coded as a CES but, if using the default GTAP assumptions, uses a Leontief structure for investment and a Cobb-Douglas for government expenditures.

5. There is additional flexibility regarding factor mobility. All factors are assumed to have a CET allocation function with the possibility of perfect transformation. An additional specification allows for sector-specific factors, for example for natural resources.

6. Aggregate factor supply is explicitly modeled with the possibility of an upward sloping supply curve.

7. Allocation of domestic output across destination markets is specified using a nested CET structure, analogously to the CES specification of the allocation of domestic absorption. The specification allows for perfect transformation, the explicit assumption in the GTAP model.

8. The supply of international trade and transport margins allows for a CES cost structure—the GTAP default is Cobb-Douglas. The sector is also assumed to be an Armington agent—though given the structure of the input database, Armington demand is equivalent to assuming all supply is sourced domestically.

9. To simplify coding, there is a single set of Armington agents that includes firms, private and public expenditures, investment expenditures and supply of international trade and transport margins. The top-level Armington decomposition across agents then only needs a single module.

10. There is somewhat more flexibility in many of the key parameters that are indexed by both region and sector.

[1] There are of course many other GAMS-based models that rely on the GTAP database. A partial list can be found at `www.gtap.agecon.purdue.edu/about/data\_models.asp`.

This document is not intended as an introduction to the GTAP model. Potential users are strongly urged to consult Hertel [1997], particularly Chapter 2, McDougall [2003] for a description of the representative household module, Brockmeier [2001] for a graphical description of the GTAP model and McDonald and Thierfelder [2004] for a description of the SAM structure of the underlying database. The description of the separate modules of the model provide some guide to the linkages between the GAMS and GEMPACK nomenclature. The GAMS code provides some guidance to the links across model specifications regarding the equation nomenclature.

The next section provides a snapshot of the GTAP model, i.e. its main features. That is followed by a full and detailed description of each module of the model. It takes a somewhat different approach from the GTAP description. The main approach is that of the circular flow of the economy. It starts with production that generates income. Income is distributed across different agents, for example private and public expenditures and savings. The demand modules follow, describing demand at the so-called Armington level. This is followed by the trade section—the allocation of Armington demand across different regions of origin, and the allocation of domestic supply across different regions of destination. There is a short section describing the demand and supply of international trade and transport services. Equilibrium on the goods and factor markets are subsequently described. And a section on investment behavior and closure finishes the description of the static model. A final module describes some potential dynamic elements of the model including changes in technology and preferences.

The final section describes the accounting framework of the model. Two accounting frameworks will be described. The first is an analytical SAM as derived from model results. The second describes the correspondence between the GTAP database and the variables from the GAMS model. The SAM is not a standard SAM and does not represent the full functionality of the underlying database. For example, demand is specified at the Armington level. This is to keep the size of the SAM at a reasonable level and yet illustrate the main accounting relations. Auxiliary matrices are described that complete the full accounting system.

A future version of this paper will include a more thorough description of model diagnostics and decomposition of results as provided in the GEMPACK version.

2 The standard GTAP model in a nutshell

The standard GTAP model is a fairly straightforward comparative static global computable general equilibrium model. It is multi-sectoral—with up to 57 sectors[2]—and multi-region—with up to 140 regions.[3] Each country module has an identical specification (i.e. the GTAP model uses a template model for each country), though with country specific parameterization that depends on the underlying database (largely an input/output table) and country-specific key parameters such as supply and demand elasticities.

The country models are linked through two sets of relations. The first set is trade and the GTAP model traces the flow of bilateral trade between any pair of regions for all sectors. Each bilateral trade node is also identified with four sets of prices—the producer price of exports, the border price of exports (or the FOB price) that incorporates export taxes and subsidies, the border price of imports (or the CIF price) that incorporates international trade and transport margins and the domestic post-border price of imports that incorporates tariff measures (but excludes domestic end-user taxes on imports). The GTAP model also includes some measure of international capital flows that respond to relative changes in anticipated rates of returns to capital across regions.

[2] Through version 9 of the GTAP data release.
[3] Through version 9 of the GTAP data release.

The supply side for each country model is based on a nested structure of constant-elasticity-of-substitution (CES) functions to represent the key substitution across inputs—both intermediate goods and factors of production. In the standard GTAP model, intermediate goods are assumed to be strictly proportional to output, i.e. a Leontief technology is assumed, though with the possibility of substitution between domestic and imported goods. There is a single CES nest for factor inputs, or endowments, which include capital, land (in agriculture), a natural resource endowment (in some sectors such as fossil fuels and forestry) and labor, of which there can be several types.[4]

Factor income and revenues generated by taxes are all allocated to a single representative household for each region. The representative household is endowed with a nested structure of preference functions to allocate regional income between demand for goods and services and savings. The top nest is a Cobb-Douglas preference function that allocates income between private consumption, public (or government) consumption, and regional savings. Private consumption is allocated across goods and services using a constant-difference-in-elasticity (CDE) utility function that is non-homethetic and allows for relatively flexible price response. Government expenditure is allocated across goods and services using a CES preference function (with a default CES elasticity of 1). Regional investment expenditures—identified as a separate agent in the GAMS version of GTAP[5]—are similarly allocated using a CES preference function (with a default CES elasticity of 0). Regional investment is equal to regional savings adjusted by international capital flows further described below.

Each agent's demand for goods and services is first specified at the so-called Armington level, i.e. a composite commodity that includes both domestically produced and imported goods.[6] Each agent then decomposes the demand for the composite bundle into demand for a domestically produced good and an (aggregate) imported good using a CES preference function. The sum across agents of the former is then equated to domestic supply of domestic goods to derive the equilibrium price of domestically produced goods sold to the domestic market. Aggregate import demand needs to be allocated across source regions. In the standard GTAP model this is done by an aggregate import agent. Thus import demand (for the composite import good) is summed across all agents, and this aggregate import demand is allocated across all sourcing regions using a CES preference function.[7]

Factor supplies are assumed to be exogenous in the standard GTAP model. The formulation herein allows for upward sloping supply curves, albeit the default elasticities are zero. All factor supplies are assumed to be partially or fully mobile across production activities—the former are sometimes referred to as sluggish factors. The degree of mobility is captured with a constant-elasticity-of-transformation (CET) supply function. For natural resources, the CET elasticity is set very low to mimic zero mobility. The version of the GTAP model described herein allows for sector-specific factor supply functions that can be used for the natural resource factor.

International capital flows are captured with a construct called the *global bank*. The latter collects savings across all regions and re-allocates these savings in a way that captures relative changes in the expected rates of returns across regions. This is more fully described below.

Like most CGE models, GTAP incorporates a very wide range of price wedges—mostly in the form of taxes and subsidies. All of these are fully detailed below. As well, the GTAP model incorporates a wide range of levers for technology and preference changes—widely suited for both

4 Most recent releases have had two types of labor—skilled and unskilled. However the latest release has five labor categories.

5 In the GEMPACK version of the model, the investment agent is incorporated with other production activities, though using no factor endowments.

6 Armington [1969] in a seminal paper described import demand using a differentiated goods model.

7 There is considerable ongoing research to make the second CES nest be agent specific, i.e. each agent in the economy would source by region directly. This literature is referred to as multi-regional input-output or MRIO.

comparative static and dynamic analysis.[8]

3 Model specification in GAMS

3.1 Set definitions

This section describes the principle sets used in the description of the model and also correspond to those used in the GAMS code. Table 1 provides the key indices and sets using the model description. The following are a few additional notes in reference to the table:

1. The code differentiates between activities (i.e. production), indexed by a, and commodities indexed by i. Unlike in the GTAP model, the activity linked to the label `CGDS` is separately identified as the 'investment agent' and has a standard final demand expenditure function. This is the usual treatment in many CGE models.

2. The domestic supplier of international trade and transport margins is treated as an Armington agent. This is an extension of the standard GTAP model, however, with no impact as the Armington demand is *de facto* equated to domestically sourced goods. This implementation is largely for convenience as it allows for collapsing many of the final demand equations using a generic specification.

3. Almost all model equations are indexed by r which is the regional dimension. Bilateral variables need two regional indices. The first is always the region of origin, or source region. The second is always the destination region. For transparency, the regional index s is used in describing the import demand specification, to reflect the *source* or exporting region. The regional index d is used in the export supply specification to reflect the *destination* or importing region.

4. The allocation of factors of production relies on the segmentation of factors that are partially or fully mobile fm and those that are sector-specific fnm. Using this definition, all factors in the GEMPACK implementation of the model are in the mobile subset fm, because even natural resources are specified using the CET allocation mechanism, albeit with an extremely small transformation elasticity. The GAMS version of the code does not differentiate between sluggish and perfectly mobile factors. This is driven instead by the input parameters. If the transformation elasticity is ∞, then the model will ensure that the law-of-one-price holds and allows for perfect factor mobility.[9] The GAMS version extends the GTAP version in that it allows for sector-specific factors (such as natural resources) that have their own potentially upwards sloping supply curve.

3.2 Production

Production is implemented as a series of nested CES functions. The main objective is to capture the key substitution and complementary relations across the various inputs. In the standard GTAP

[8] The current GAMS version of the model only incorporates some of these features.

[9] GAMS will interpret the value `INF` as infinity and allows limited operations with this value—in particular testing in logical expressions.

Table 1: Sets used in model specification

Set	Alias	Description
i		Commodities
a		Activities
h		Household (or private) agent
gov		Government (or public) agent
inv		Investment agent
tmg		Domestic supplier of margins
aa		Armington agents $= \cup\{a, h, gov, inv, tmg\}$
$fd(aa)$		Final demand accounts $= \cup\{h, gov, inv, tmg\}$
$m(i)$		International trade and transport services
f		Factors of production
$l(f)$		Labor types (e.g. unskilled and skilled)
$cap(f)$		Capital factor
$lnd(f)$		Land factor
$nr(f)$		Natural resource factor
$fm(f)$		Partially or fully mobile factors
$fnm(f)$		Sector-specific factors
r	s, d	Regions of the model
$HIC(r)$		High-income regions
$MANU(i)$		Manufactured commodities

database the inputs include intermediate goods and services, indexed by i for each activity a,[10] and factors of production, indexed by f for each activity a. The production nest is depicted graphically in figure 1.

The top CES nest consists of two bundles. The first composite bundle is an aggregate of the factors of production, i.e. the value added bundle (VA). The second bundle is an aggregate of intermediate demand (ND).[11] Equations (1) and (2) represent the CES derived demand functions for the two bundles, respectively VA and ND. The bundle prices are PVA and PND respectively, where PX represents the aggregate price of the two bundles. Given the assumption of constant-returns-to-scale the price PX is also the unit cost of production. The parameters α^{va} and α^{nd} are the standard CES (dual) share parameters. There are three technology coefficients. A^{xp} represents a uniform shifter, whereas λ^{va} and λ^{nd} are bundle specific shifters. The CES substitution elasticity is given by σ^p.[12]

$$VA_{r,a} = \alpha^{va}_{r,a} XP_{r,a} \left(\frac{PX_{r,a}}{PVA_{r,a}}\right)^{\sigma^p_{r,a}} \left(A^{xp}_{r,a}\lambda^{va}_{r,a}\right)^{\sigma^p_{r,a}-1} \qquad (1)$$

[10] Unlike in the GTAP data and model, the index a does not include the investment good (identified with the label `CGDS`). The investment good is treated like the other final demand components with its own individually identified expenditure function. By default, the expenditure function is a CES function with zero elasticity, which is the GTAP default.

[11] In the standard GTAP model, there is no aggregate intermediate demand bundle. The top nest consists of the value added bundle and all intermediate goods taken individually.

[12] The substitution elasticity in GTAP is given by the parameter `ESUBT`. It is invariably set to zero. Note as well that in the GTAP model it is not region specific.

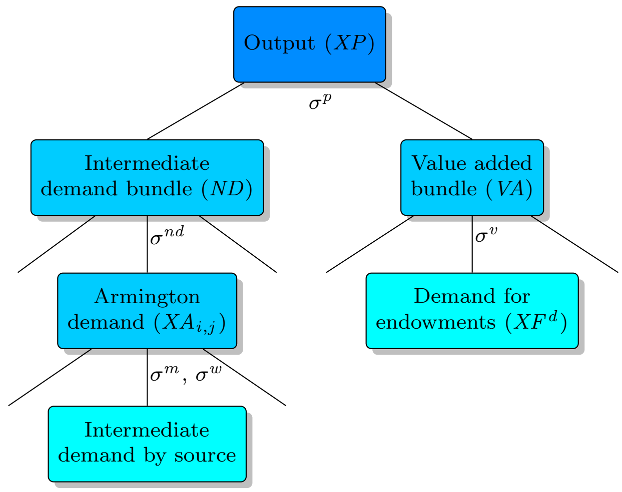

Figure 1: **Production CES nest**

$$ND_{r,a} = \alpha^{nd}_{r,a} XP_{r,a} \left(\frac{PX_{r,a}}{PND_{r,a}}\right)^{\sigma^p_{r,a}} \left(A^{xp}_{r,a}\lambda^{nd}_{r,a}\right)^{\sigma^p_{r,a}-1} \tag{2}$$

Equation (3) represents the zero profit condition and hence determines the unit cost of production, PX. A wedge between the unit cost and market price is introduced and represents the *ad valorem* production (or output) tax, τ^p.[13] The market price of output, PP, is given by equation (4).

$$PX_{r,a}XP_{r,a} = PVA_{r,a}VA_{r,a} + PND_{r,a}ND_{r,a} \tag{3}$$

$$PP_{r,a} = PX_{r,a}(1 + \tau^p_{r,a}) \tag{4}$$

This subunit determines VA, ND, PX and PP that correspond to variables `qva`, `ps` and `pm` in the TABLO version[14] of the GTAP model.[15]

Taxes and subsidies

The GAMS and GEMPACK versions handle some of the tax and subsidy instruments in different ways. First, most of the GEMPACK taxes and subsidies are handled using so-called power variables. Thus, the output, or market price equation in the GEMPACK formation would be:

$$PP = T.PX$$

where T is defined as:

$$T = (1 + \tau^p)$$

[13] In the GTAP model, the output tax is introduced relative to the output price, not with respect to the unit cost. Hence a production subsidy is positive in GTAP, but is negative in this version of the model.

[14] Note that most of the variables in the GEMPACK version of the GTAP model represent percent changes of the relevant variable—where the use of lower case variable names is typically understood to represent percent changes. Upper case names typically represent so-called coefficients and are either initialized at the beginning of a model simulation or updated between solution iterations.

[15] The GTAP model has no direct correspondence to the variable ND.

Second, many subsidies and taxes are measured relative to the market price and subsidies are expressed as a positive wedge with respect to the market price. The output price equation is then written as:

$$PP(1+\varsigma) = PP.TO = PX$$

where ς is positive for a subsidy and negative for a tax. The power variable TO is thus greater than 1 for a subsidy. The linearization of the last expression leads to:

$$\dot{PP} + \dot{TO} = \dot{PX}$$

which is equivalent to GEMPACK's `OUTPUTPRICES` equation `ps(i,r) = to(i,r) + pm(i,r);`. The two instruments are linked via the following expression:

$$\tau^p = -\frac{\varsigma}{1+\varsigma} \iff \varsigma = -\frac{\tau^p}{1+\tau^p}$$

In the GAMS implementation, the price wedge is negative for a subsidy and positive for tax. The revenue formulas, which are evaluated as less than 0 for subsidies, are given by:

$$R = \tau^p.PX.XP = -\varsigma.PP.XP$$

A shock of Δ to τ^p such that:

$$\tau_1^p = \tau_0^p + \Delta$$

translates into the following shocks in the GEMPACK formulation:

$$TO_1 = \frac{TO_0}{1+\Delta TO_0}$$
$$\dot{TO} = -\frac{\Delta TO_0}{1+\Delta TO_0}$$
$$\delta = \varsigma_1 - \varsigma_0 = -\frac{\Delta TO_0^2}{1+\Delta TO_0}$$

On the other hand, a percent shock to the GAMS instrument, such that:

$$\tau_1^p = \tau_0^p(1+\chi)$$

is equivalent to the following formulas for the relevant GEMPACK formulas:

$$TO_1 = \frac{TO_0}{1-\chi(TO_0-1)}$$
$$\dot{TO} = \frac{\chi(TO_0-1)}{1-\chi(TO_0-1)}$$
$$\beta = \frac{\varsigma_1}{\varsigma_0} - 1 = \frac{\chi TO_0}{1-\chi(TO_0-1)}$$

When χ is -1, i.e. a complete removal of the wedge, the top formula simplifies to 1 and the third formula simplifies to -1.

[TO BE CONFIRMED] All taxes in GEMPACK are handled using variables expressed as powers. One advantage of this in GEMPACK is that they can be expressed as percent changes since the power variables are normally centered around 1 whereas the tax rates are centered around 0 and thus require to be handled as ordinary changes and not percent changes. Taxes that are specified relative to the market price such as described above include the output or production taxes (τ^p or `to`), income taxes (κ^f or `to`) and export taxes (τ^e or `txs` and `tx`). All other taxes are expressed the same way in both versions of the model.

The following nest decomposes the aggregate value added bundle, *VA*, into its components, i.e. the various factors of production. A single nest is used in the standard GTAP model, though many of the GTAP variants allow for a more complex nesting—for example the isolation of the land factor in the agricultural sectors. Equation (5) determines the derived demand for factor f,[16] XF^d, where the CES substitution elasticity across factors is given by σ^v.[17] The purchasers' (or agents') price of factors is given by PF^a. The parameter α^f represents the standard CES (dual) share parameter. Factor specific technology shifters are given by λ^f. Equation (6) determines the aggregate value added price, PVA, using the CES dual price aggregation expression.

$$XF^d_{r,f,a} = \alpha^f_{r,f,a} VA_{r,a} \left(\frac{PVA_{r,a}}{PF^a_{r,f,a}}\right)^{\sigma^v_{r,a}} \left(\lambda^f_{r,f,a}\right)^{\sigma^v_{r,a}-1} \tag{5}$$

$$PVA_{r,a} = \left[\sum_f \alpha^f_{r,f,a} \left(\frac{PF^a_{r,f,a}}{\lambda^f_{r,f,a}}\right)^{1-\sigma^v_{r,a}}\right]^{1/(1-\sigma^v_{r,a})} \tag{6}$$

This subunit determines XF^d and PVA corresponding to variables `qfe` and `pva` in the GTAP code.

The next set of equations relate to the intermediate demand nest of the model. It decomposes the *ND* bundle into demand for goods and services by sector. Equation (7) determines intermediate demand by commodity (at the Armington level), *XA*, with (Armington) prices given by *PA*.[18] The substitution elasticity is given by σ^{nd} and is typically 0.[19] Technology shifters are provided by the λ^{io} coefficients. Equation (8) determines the price of the aggregate intermediate demand bundle *PND* using the CES dual price expression.

$$XA_{r,i,a} = \alpha^{io}_{r,i,a} ND_{r,a} \left(\frac{PND_{r,a}}{PA_{r,i,a}}\right)^{\sigma^{nd}_{r,a}} \left(\lambda^{io}_{r,i,a}\right)^{\sigma^{nd}_{r,a}-1} \tag{7}$$

$$PND_{r,a} = \left[\sum_i \alpha^{io}_{r,i,a} \left(\frac{PA_{r,i,a}}{\lambda^{io}_{r,i,a}}\right)^{1-\sigma^{nd}_{r,a}}\right]^{1/(1-\sigma^{nd}_{r,a})} \tag{8}$$

This subunit determines $XA_{i,a}$ and *PND*. The former corresponds to variable `qf` in the GTAP code.[20]

The decomposition of the Armington demand will be discussed below in the trade section. It is consolidated for all Armington agents, unlike the GEMPACK code that has it specified for each Armington agent.

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16 The factors are indexed by i in the GEMPACK code, though signaling that i covers the endowment commodities.

17 The substitution elasticity in GTAP is given by `ESUBVA` and is not region specific.

18 The Armington variables and all of their components are indexed by i, i.e. commodity, and aa that indexes all of the Armington agents. The Armington agents include all firms (or activities) indexed by a, households indexed by h, the government sector indexed by gov, the investment sector indexed by inv and trade margins, indexed by tmg. The latter is for convenience only as the data assumes that all exported international trade and transport services are sourced within the exporting country.

19 This equation has no direct equivalent in the GTAP code where intermediate demand is directly linked to the level of output, with the possibility of a non-zero substitution elasticity given by `ESUBT`.

20 The variable *PND* has no correspondence in the GEMPACK version of the model as intermediate demand, i.e. `qf`, is expressed directly in relation to output and not to an intermediate demand bundle.

3.3 Commodity supply

The GAMS version of GTAP distinguishes between activities and commodities. A 'make' matrix is used to convert output of activity a into one or more commodities indexed by i. In other words, activities can produce one or more commodities, for example the ethanol sector could produce both ethanal and DDGS. A CET specification is used to allocate the production of activity a into supply of its various commodities. Similarly, a national agent buys various commodities labeled i produced by one or more activities to provide a national, or aggregate, supply of good i. For example a national electricity supplier could buy electricity from different power generators—thermal, nuclear, hydro, renewables, etc. A CES specification is used to aggregate output from one or more activities.

Equation (9) represents the supply of commodity i produced by activity a, $X_{a,i}$, derived from a standard CET specification. In the standard GTAP model, the matrix X will be diagonal, i.e. each activity produces one and only one commodity.[21] In this formulation, each activity can produce one or more commodities. If the commodities being produced are homogeneous, i.e. the transformation elasticity given by ω^s is infinite, the law-of-one-price holds. Equation (10) represents the zero profit condition.

$$\begin{cases} X_{r,a,i} = \gamma^x_{r,a,i} XP_{r,a} \left(\dfrac{P_{r,a,i}}{PP_{r,a}}\right)^{\omega^s_{r,a}} & \text{if } \omega^s_{r,a} \neq \infty \\ P_{r,a,i} = PP_{r,a} & \text{if } \omega^s_{r,a} = \infty \end{cases} \tag{9}$$

$$PP_{r,a} XP_{r,a} = \sum_i P_{r,a,i} X_{r,a,i} \tag{10}$$

In an analogous fashion, equation (11) represents the demand side of the 'make' matrix. A national buyer of commodity i purchases goods from the different national producers (indexed by a) using a CES preference function. National supply of commodity i is represented by the variable XS. It eventually will be allocated to domestic and export markets (see below). The specification allows for commodity homogeneity if the substitution elasticity, σ^s, is infinite. Equation (12) represents the standard zero-profit condition.

$$\begin{cases} X_{r,a,i} = \alpha^x_{r,a,i} XS_{r,i} \left(\dfrac{PS_{r,i}}{P_{r,a,i}}\right)^{\sigma^s_{r,i}} & \text{if } \sigma^s_{r,i} \neq \infty \\ P_{r,a,i} = PS_{r,i} & \text{if } \sigma^s_{r,i} = \infty \end{cases} \tag{11}$$

$$PS_{r,i} XS_{r,i} = \sum_a P_{r,a,i} X_{r,a,i} \tag{12}$$

This module determines X, P, XP and PS. There is no corresponding module in the standard GTAP model. However, if the 'make' matrix is diagonal, XP and PS correspond respectively to `qo` and `pm` in the GTAP code.[22]

[21] Though the standard GTAP database has a diagonal 'make' matrix, it is possible to make it non-diagonal by providing two separate mappings for produced and consumed commodities. For example, one could aggregate all agricultural production into a single activity, but have it produce a variety of agricultural commodities.

[22] In the case of diagonality, $XP = XS$ and $PP = PS$ if units are identical. The 'make' matrix could be used to convert from one system of units to another, for example from dollars to megawatt hours.

3.4 Income distribution

There are two main sources of income. The first is tax revenues generated by the myriad of taxes and subsidies. The second is the revenues generated by the use of factors in the production of goods. Income is then allocated between private and public expenditure and savings.

Tax revenues are generated by taxes on production inputs (both intermediate goods and factors of production), production taxes, sales taxes on both domestic and import consumption, import and export taxes, and direct taxes on factor income. Equation (13) defines revenues from production (or output) taxes.[23] Equation (14) defines tax revenues generated by taxes on commodity sales across all Armington agents. Commodity taxes are allowed to differ between goods sourced domestically and imported. Equations (15) and (16) define tax revenues generated by taxes and subsidies on factors of production. The price PF represents the market (or equilibrium) price of factors. Equation (17) defines taxes on imports, where τ^m represents the bilateral tariff rates applied to imports from s into r and PM^{CIF} is the border or CIF price of imports. The variable XW represents bilateral trade flows. The first regional index is the exporting region and the second regional index is the importing region. The index s will be used to indicate the *source* country when used to describe imports and the index d will be used to indicate the *destination* region when used to describe exports. Equation (18) defines taxes/subsidies on exports, where τ^e represents the bilateral tax/subsidy applied to exports from r imported by d. The tax/subsidy is applied to the producer price of exports, PE.[24] Equation (19) represents the tax revenues generated by direct taxes on factor income where κ^f is the tax rate, and the product $PFT_{r,f}XFT_{r,f}$ represents the aggregate factor remuneration for factor f.

$$YTAX_{r,pt} = \sum_a \tau^p_{r,a} PX_{r,a} XP_{r,a} \tag{13}$$

$$YTAX_{r,at} = \sum_{aa} \sum_i \left[\left(\tau^{dtx}_{r,i,aa} + \chi^{tx}_{r,aa} \right) PD_{r,i} XD_{r,i,aa} + \left(\tau^{mtx}_{r,i,aa} + \chi^{tx}_{r,aa} \right) PMT_{r,i} XM_{r,i,aa} \right] \tag{14}$$

$$YTAX_{r,ft} = \sum_a \sum_f \left[\tau^{ft}_{r,f,a} PF_{r,f,a} XF^d_{r,f,a} \right] \tag{15}$$

$$YTAX_{r,fs} = \sum_a \sum_f \left[\tau^{fs}_{r,f,a} PF_{r,f,a} XF^d_{r,f,a} \right] \tag{16}$$

$$YTAX_{r,mt} = \sum_i \sum_s \left[\left(\tau^m_{s,i,r} + \zeta^m_{r,i} \right) PM^{CIF}_{s,i,r} XW_{s,i,r} \right] \tag{17}$$

$$YTAX_{r,et} = \sum_i \sum_d \left[\left(\tau^e_{r,i,d} + \zeta^e_{r,i} \right) PE_{r,i,d} XW_{r,i,d} \right] \tag{18}$$

$$YTAX_{r,dt} = \sum_f \left[\kappa^f_{r,f} PFT_{r,f} XFT_{r,f} \right] \tag{19}$$

This subunit generates $YTAX$ with its various components. In the GEMPACK version of GTAP, the corresponding variables represent the (ordinary) change in the respective revenue stream and not the percent change. These variables are `del_taxrout` (output tax), `del_taxrgc` (tax on

[23] The tax is applied on the unit cost of production, equivalent to the farm- or factory-gate price. In the standard GTAP model, the tax is applied to the market price.

[24] In the GTAP model, export taxes are applied to the border price.

government consumption), `del_taxrpc` (tax on private consumption), `del_taxriu` (tax on intermediate consumption and investment goods), `del_taxrfu` (tax on factor inputs), `del_taxrimp` (tax on imports), `del_taxrexp` (tax on exports) and `del_taxrinc` (direct taxes on factor income).[25]

Total revenues from taxes is defined in equation (20), where the sum is over all tax revenue streams as defined by the index gy. Total revenues from indirect taxes is equal to total taxes less direct taxes, equation (21).

$$YTaxTot_r = \sum_{gy} YTAX_{r,gy} \tag{20}$$

$$YTaxInd_r = YTaxTot_r - YTAX_{r,dt} \tag{21}$$

This subunit calculates $YTaxTot$ and $YTaxInd$. These variables correspond to the GEMPACK version variables `del_ttaxr` and `del_indtaxr`.

Equation (22) represents total factor income net of depreciation. Note that factor income is defined at market prices, not net of direct taxes. The variable PI represents the unit cost of investing, i.e. the replacement cost of capital goods, K^0 is the beginning of period capital stock and δ is the depreciation rate. Total regional income, Y, is defined in equation (23) and is equal to the sum of factor income net of depreciation and total revenues generated by indirect taxes.

$$factY_r = \sum_f \left[PFT_{r,f} XFT_{r,f}\right] - \delta_r PI_r K_r^0 \tag{22}$$

$$Y_r = factY_r + YTaxInd_r \tag{23}$$

This subunit calculates $factY$ and Y corresponding to the GEMPACK variables `fincome` and `y`.

3.5 Allocation of regional income

There are three domestic final demand agents—private households, government and investment. Their demand is specified with a nested preference structure that first allocates total regional income to the three agents and then each agent has an agent-specific preference function that determines the demand for goods and services. The nested demand structure is depicted in figure 2.[26]

Regional income, Y, is allocated across three agents using a top level Cobb-Douglas utility function. The three aggregate expenditure categories are private and public expenditures and aggregate savings. The representative regional household is assumed to maximize utility according to the following scheme:

$$\max U_r = A_r^u {U_r^P}^{\beta_r^P} {U_r^G}^{\beta_r^G} {U_r^S}^{\beta_r^S}$$

subject to

$$Y_r = E_r^P(U_r^P, P_r^P) + E_r^G(U_r^G, P_r^G) + E_r^S(U_r^S, P_r^S)$$

[25] The standard GTAP model does not make use of factor subsidies (data contained in the array `FBEP` in the GTAP data base).

[26] The graphic depicts a situation excluding international capital flows. The top level Cobb-Douglas preference structure determines the level of domestic savings. The level of domestic investment will differ to the extent the region attracts savings from abroad, or vice-versa.

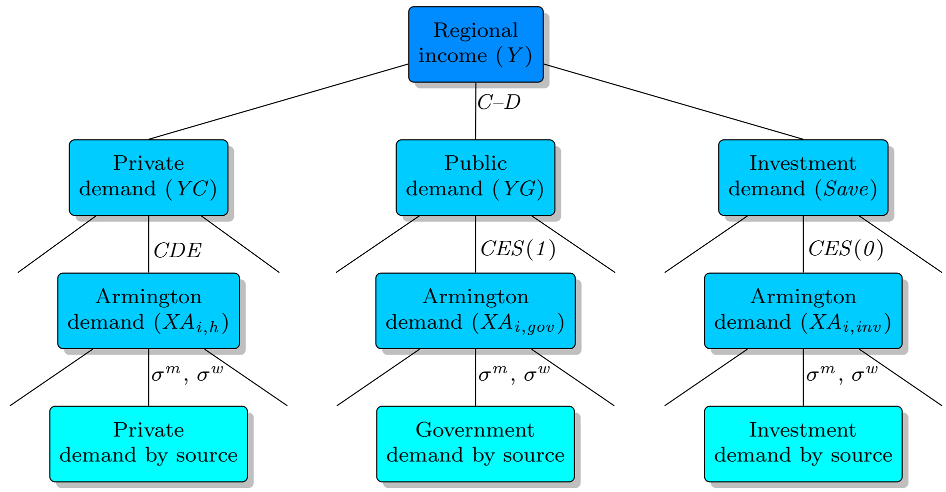

Figure 2: **Demand nest**

where the superscript indices refer respectively to private or household consumption (P), government or public consumption (G) and savings (S). The expenditure functions for government and savings are both of a generic CES variety and thus the expenditure function can be written as $E = Auf(P)$, where $f(P)$ in this case is the CES dual price expression. The household expenditure function is based on a CDE utility function and has no simple expression. The derivation of the expenditure shares requires expressions for the elasticity of total expenditure with respect to total utility, Φ, that in turn requires the elasticity of private expenditure with respect to private utility, (see McDougall [2003]). Equation (24) determines the latter. For the CDE expenditure function, the elasticity of expenditure with respect to utility is the weighted sum of the CDE expansion parameters (e), where the weights are given by the (private consumption) budget shares (s_i^p).[27] The total expenditure elasticity (with respect to total utility) is given in equation (25) and is the inverse of the sum of the individual Cobb-Douglas share parameters with weights given by the inverse of the individual expenditure elasticities.

$$\phi_r^P = \sum_i s_{r,i}^p e_{r,i} \tag{24}$$

$$\Phi_r = \left[\beta_r^P / \phi_r^P + \beta_r^G + \beta_r^S\right]^{-1} \tag{25}$$

The expenditures are derived from utility maximization and are given in equations (26) through (28). The equations determine respectively aggregate private consumption (YC), aggregate public consumption (YG) and total regional savings ($Save$). The expenditures are provided at aggregate level since regional income (Y) is aggregate regional income. Per capita levels can be determined by dividing through by population.

$$YC_r = \beta_r^P \frac{\Phi_r}{\phi_r^P} Y_r \tag{26}$$

$$YG_r = \beta_r^G \Phi_r Y_r \tag{27}$$

[27] The expenditure elasticity for the other two expenditure functions is 1 due to the form of the expenditure function, i.e. the use of a CES function.

$$Save_r = \beta_r^S \Phi_r Y_r \tag{28}$$

This subunit derives ϕ^P, Φ, YC, YG, and $Save$. The corresponding variables in the GEMPACK version are `uepriv`, `uelas`, `yp`, `yg` and `qsave`.

3.6 Utility of representative household

The top level utility function depends on the utility of the sub-components. Equation (29) defines (implicitly) utility from private consumption based on the CDE utility function, U^P. It is exclusively a function of consumer prices and per capita private expenditure and the parameters of the utility function. The CDE function is more fully described in Hertel [1997]. The e parameters are known as the expansion parameters and are linked to the income elasticities. The b parameters are the substitution parameters. The share parameters (α^a) and the consumer (Armington) prices are both indexed by h, which is an index in the set of Armington agents. Utility is per capita as aggregate private expenditure is divided by total population. Utility from public expenditure, U^G, and savings, U^S, are provided in equations (30) and (31) respectively, where XG is the aggregate volume of public spending and XS is equal to nominal savings divided by the price (index) of savings. These functional forms can be derived from the generic CES expenditure function. Equation (32) defines total (per capita) utility, U.

$$\sum_i \alpha^a_{r,i,h} PA^{b_{r,i}}_{r,i,h} \left(U^P_r\right)^{b_{r,i}e_{r,i}} \left(\frac{YC_r}{Pop_r}\right)^{-b_{r,i}} \equiv 1 \tag{29}$$

$$U^G_r = A^{ug}_r \frac{XG_r}{Pop_r} \tag{30}$$

$$U^S_r = A^{us}_r \frac{Save_r/PSave_r}{Pop_r} \tag{31}$$

$$U_r = A^u_r U^{P^{\beta^P_r}}_r U^{G^{\beta^G_r}}_r U^{S^{\beta^S_r}}_r \tag{32}$$

This subunit generates U^P, U^G, U^S and U corresponding to the GEMPACK variables `up`, `ug`, `dpsave`[28] and `u`.

3.7 Private consumption

Consumer demand as derived from the CDE utility function is defined by equation (33). The ratio defines per capita consumption that is then multiplied by population to derive aggregate private consumption, XA. The latter is part of the Armington matrix that covers all Armington agents.

$$XA_{r,i,h} = Pop_r \frac{\alpha^a_{r,i,h} b_{r,i} PA^{(b_{r,i}-1)}_{r,i,h} \left(U^P_r\right)^{b_{r,i}e_{r,i}} \left(\dfrac{YC_r}{Pop_r}\right)^{(1-b_{r,i})}}{\sum_j \alpha^a_{r,j,h} b_{r,j} PA^{b_{r,j}}_{r,j,h} \left(U^P_r\right)^{b_{r,j}e_{r,j}} \left(\dfrac{YC_r}{Pop_r}\right)^{b_{r,j}}} \tag{33}$$

If we define the following auxiliary variable:

[28] TO BE VERIFIED

$$Z_{r,i} = \alpha^a_{r,i,h} PA^{b_{r,i}}_{r,i,h} \left(U^P_r\right)^{b_{r,i} e_{r,i}} \left(\frac{YC_r}{Pop_r}\right)^{-b_{r,i}}$$

The expression for consumption can be simplified to:

$$XA_{r,i,h} = \frac{b_{r,i} Z_{r,i} \dfrac{YC_r}{PA_{r,i,h}}}{\displaystyle\sum_j b_{r,j} Z_{r,j}}$$

The private consumption budget shares, s^p, are defined in equation (34). Equation (35) provides one definition of the consumer price index, PC.

$$s^p_{r,i} = \frac{PA_{r,i,h} XA_{r,i,h}}{YC_r} \quad (34)$$

$$PC_r = \sum_i s^p_{r,i} PA_{r,i,h} \quad (35)$$

This subunit generates the variables $XA_{r,i,h}$, s^p and PC corresponding to the GEMPACK variables `qp` (private consumption) and `ppriv` (consumer price index). The budget shares in GEMPACK (`CONSHR`) are treated as coefficients in the code and are updated at each iteration.

3.8 Government consumption

The allocation of aggregate government expenditure across goods and services uses a CES expenditure function.[29] Equation (36) determines the *gov* vector in Armington demand, XA.

$$XA_{r,i,gov} = \alpha^a_{r,i,gov} XG_r \left(\frac{PG_r}{PA_{r,i,gov}}\right)^{\sigma^g_r} \quad (36)$$

The government expenditure price deflator, PG, is defined in equation (37).[30]

$$PG_r = \left[\sum_i \alpha^a_{r,i,gov} PA^{(1-\sigma^g_r)}_{r,i,gov}\right]^{1/(1-\sigma^g_r)} \quad (37)$$

The volume of government expenditure, XG, is defined in equation (38).

$$YG_r = PG_r XG_r \quad (38)$$

This subunit determines the variables $XA_{r,i,gov}$, PG and XG corresponding to GEMPACK variables `qg` (government purchases of goods and services) and `pgov` (government expenditure price deflator).[31]

[29] The GEMPACK code assumes by default a Cobb-Douglas expenditure function, with a CES substitution elasticity of 1. The GAMS code allows for more flexibility though the default elasticity is 1.

[30] The code uses the Cobb-Douglas dual price expression if the CES elasticity is 1.

[31] TO BE VERIFIED The GEMPACK code does not explicitly use the aggregate volume of government expenditure, replacing its use by the expression $yg - pgov$ wherever it would be needed.

3.9 Investment expenditure

The GAMS code explicitly identifies the investment sector that is included in the *firm* definition of the GEMPACK code, where the investment sector is identified with the set label `CGDS`. The GAMS code allows for a generic CES expenditure function. The GEMPACK code also allows for a generic CES expenditure function, but the default substitution elasticity is given by `ESUBT` and is set to 0. Equation (39) determines the *inv* vector in Armington demand, XA. The specification allows for technological changes as measured by the variable λ^i.

$$XA_{r,i,inv} = \alpha^a_{r,i,inv} XI_r \left(\lambda^i_{r,i}\right)^{\sigma^i_r - 1} \left(\frac{PI_r}{PA_{r,i,inv}}\right)^{\sigma^i_r} \tag{39}$$

The investment expenditure price deflator, PI, is defined in equation (40).[32]

$$PI_r = \left[\sum_i \alpha^a_{r,i,inv} \left(\frac{PA_{r,i,inv}}{\lambda^i_{r,i}}\right)^{(1-\sigma^i_r)}\right]^{1/(1-\sigma^i_r)} \tag{40}$$

Investment volume, XI, is defined in equation (41). The nominal level of investment will be determined by the investment closure specification described below.

$$YI_r = PI_r XI_r \tag{41}$$

This subunit determines the variables $XA_{r,i,inv}$, PI and YI corresponding to GEMPACK variables `qf(i,CGDS,r)` (investment purchases of goods and services) and `ps(CGDS,r)` (investment expenditure price deflator).[33]

3.10 Top level Armington nest

In the standard GTAP model, the top level Armington nest is done at the agents' level that determines the agents' demand for domestic and (aggregate) import goods, respectively. Armington demand for all agents has been described above—for firms, private consumers, the public sector and investment demand.[34] At this stage, the Armington demand for each agent (and commodity) is decomposed into a domestic and import component using a CES preference structure. See figure 3.

Agents' are faced with market prices given by PD and PMT, respectively for domestic and imported goods. The former represents the market price of domestically produced goods and the latter represents the price of the aggregate import bundle. The agents' prices are equal to the market prices plus an *ad valorem* tax wedge that is agent and commodity specific given by τ^{dtx} and τ^{mtx} respectively for domestic and imported goods. Equations (42) and (43) determine the purchasers' (or agents') prices for domestic and imported goods. Both equations have an additional tax shifter, χ^{tx}, that is uniform across commodities and source. This tax shifter can be used exogenously to apply a uniform shift in the relevant commodity taxes (across all agents), or could be endogenous to target some objective.

$$PD^p_{r,i,aa} = PD_{r,i}(1 + \tau^{dtx}_{r,i,aa} + \chi^{tx}_{r,aa}) \tag{42}$$

[32] The code uses the Cobb-Douglas dual price expression if the CES elasticity is 1.

[33] TO BE VERIFIED The GEMPACK code does not explicitly use the nominal value of investment.

[34] The international trade and transport services sector is also treated as an Armington agent for convenience. The GTAP database and model assume that intermediate demand for this sector is sourced exclusively from the domestic market. The Armington demand for this sector will be described below.

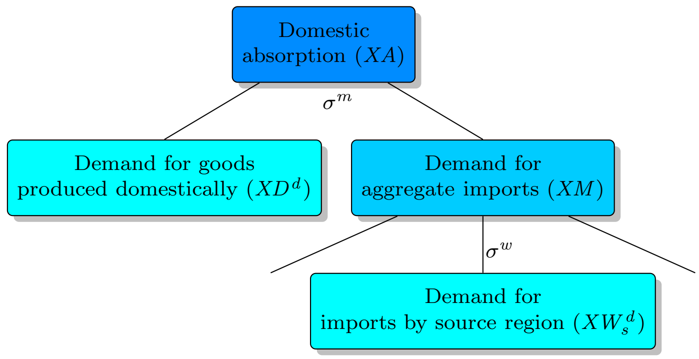

Figure 3: **Nested Armington demand**

$$PM^p_{r,i,aa} = PMT_{r,i}(1 + \tau^{mtx}_{r,i,aa} + \chi^{tx}_{r,aa}) \tag{43}$$

The Armington price for each agent, PA, is given by equation (44) that is the CES dual price expression for the aggregate price as a function of the component prices, respectively PD^p and PM^p. The Armington elasticity is given by σ^m.[35]

$$PA_{r,i,aa} = \left[\alpha^d_{r,i,aa}(PD^p_{r,i,aa})^{1-\sigma^m_{r,i}} + \alpha^m_{r,i,aa}(PM^p_{r,i,aa})^{1-\sigma^m_{r,i}}\right]^{1/(1-\sigma^m_{r,i})} \tag{44}$$

The next set of equations, (45) and (46), reflect the Armington decomposition, i.e. the demand for domestic (XD) and imported goods (XM), respectively, for each agent and for each commodity.

$$XD_{r,i,aa} = \alpha^d_{r,i,aa} XA_{r,i,aa} \left(\frac{PA_{r,i,aa}}{PD^p_{r,i,aa}}\right)^{\sigma^m_{r,i}} \tag{45}$$

$$XM_{r,i,aa} = \alpha^m_{r,i,aa} XA_{r,i,aa} \left(\frac{PA_{r,i,aa}}{PM^p_{r,i,aa}}\right)^{\sigma^m_{r,i}} \tag{46}$$

This subunit determines five Armington variables: PD^p, PM^p, PA, XD and XM. The corresponding variables in the GEMPACK code are `pfd`, `pfm`, `pf`, `qfd` and `qfm` for firms and investment, `ppd`, `ppm`, `pp`, `qpd` and `qpm` for private consumption, `pgd`, `pgm`, `pg`, `qgd` and `qgm` for private consumption, and `qst` for trade and transport margins (where the Armington assumption is used for convenience).[36]

3.11 Second level Armington nest

The second level Armington nest decomposes aggregate import demand by region of origin. In principle, this could also be done at the agent level, but for practical reasons, the second level nesting is done at the aggregate regional level, i.e. there is an aggregate importer that allocates aggregate import demand across regions of origin using a CES preference structure. Equation (47) determines aggregate import demand across all Armington agents, XMT. Equation (48) provides the allocation of aggregate imports across all source regions, indexed by s (that may eventually

[35] The top level Armington elasticity in GTAP is given by `ESUBM` that is neither region- nor agent-specific. The Armington elasticity has been made region-specific in this version, but could also be made agent-specific.

[36] The price of trade and transport margins is equal to the domestic producer price, i.e. PD in this notation, but `pm` in the GEMPACK notation where no difference is made between domestic and export markets.

include the own-region imports if the region is a combination of two or more countries). The variable XW^d represents the demand for exports from region s to region r for commodity i.[37] The variable PM is the purchasers' price of bilateral imports that is tariff inclusive (to be defined below). The formulation allows for changes in trade preferences as measured by the variable λ^m. The price of aggregate imports, PMT, is defined in equation (49) that represents the zero profit condition for import demand. It could be equivalently specified using the CES dual price expression.

$$XMT_{r,i} = \sum_{aa} XM_{r,i,aa} \tag{47}$$

$$XW^d_{s,i,r} = \alpha^w_{s,i,r} XMT_{r,i} \left(\lambda^m_{s,i,r}\right)^{\sigma^w_{r,i}-1} \left(\frac{PMT_{r,i}}{PM_{s,i,r}}\right)^{\sigma^w_{r,i}} \tag{48}$$

$$PMT_{r,i} XMT_{r,i} = \sum_{s} PM_{s,i,r} XW^d_{s,i,r} \tag{49}$$

This subunit generates the variables XMT, XW^d and PMT. The corresponding GEMPACK variables are `qim`, `qxs` and `pim`.

3.12 Allocation of domestic supply

Domestic supply, XS, is sold to the domestic (i.e. regional market) and abroad to the various regions of the model in the form of exports (including own-exports).[38] In the standard GTAP model, all output is sold at a uniform producer price that is PS in the GAMS version of the model, and `pm` in the GEMPACK version. The GAMS version of the model allows for imperfect transformation of domestic supply across various markets of destination, see figure (4). Analogously to the implementation of import demand, the allocation of domestic supply uses a nested CET structure. In the first nest, the aggregate domestic supplier allocates supply between the domestic market and an aggregate exporter. The latter in turn allocates aggregate exports across the various regions of the model thereby determining bilateral export supply.

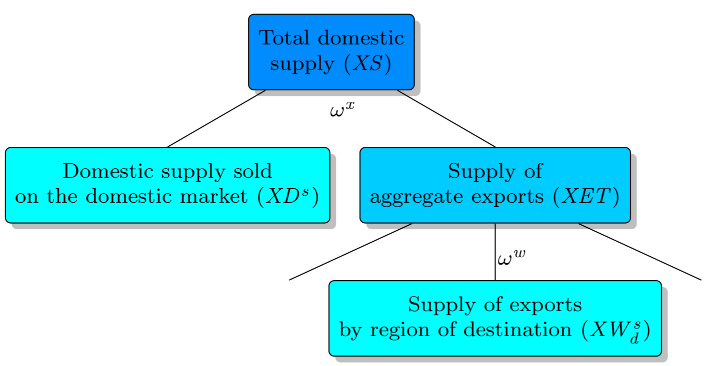

Figure 4: **Nested CET transformation of domestic output**

Equations (50) and (51) describe the top level CET supply functions for the domestic market (XD^s) and aggregate exports (XET), respectively. The formulation allows for the possibility of perfect transformation, which is the explicit assumption in the GEMPACK version of the GTAP

[37] The superscript d represents the demand for bilateral trade flows. In the model implementation, the trade equilibrium condition is substituted out.

[38] Recall that in the case of a diagonal 'make' matrix, $XS = XP$ and $PS = PP$.

model. The price PD represents the market price of domestically produced goods sold on the domestic (i.e. regional market). In the case of perfect transformation it must equal the aggregate supply price, i.e. the law-of-one-price holds. The price PET represents the price of aggregate exports, which must also equal the aggregate supply price in the case of perfect transformation. Equation (52) represents the "market clearing" condition for domestic supply. This is clearly the case with perfect transformation since the price terms can be dropped. In the case of imperfect transformation, it simply reflects the aggregation condition for domestic supply.[39] Market clearing is done at the sub-market level in the case of imperfect transformation.

$$\begin{cases} XD^s_{r,i} = \gamma^d_{r,i} XS_{r,i} \left(\dfrac{PD_{r,i}}{PS_{r,i}} \right)^{\omega^x_{r,i}} & \text{if } \omega^x_{r,i} \neq \infty \\ PD_{r,i} = PS_{r,i} & \text{if } \omega^x_{r,i} = \infty \end{cases} \tag{50}$$

$$\begin{cases} XET_{r,i} = \gamma^e_{r,i} XS_{r,i} \left(\dfrac{PET_{r,i}}{PS_{r,i}} \right)^{\omega^x_{r,i}} & \text{if } \omega^x_{r,i} \neq \infty \\ PET_{r,i} = PS_{r,i} & \text{if } \omega^x_{r,i} = \infty \end{cases} \tag{51}$$

$$PS_{r,i} XS_{r,i} = PD_{r,i} XD^s_{r,i} + PET_{r,i} XET_{r,i} \tag{52}$$

The second level CET nests allocates aggregate exports XET across the various destination export markets and hence defines bilateral export supplies. Equation (53) represents the CET supply function in the case of imperfect transformation across export markets, XW^s, that represents the exports from region r to region d for commodity i. In the case of perfect transformation, the supply function is replaced with the law-of-one-price, where the producer price of exports across all regions of destination, PE, is set equal to the producer price of domestic output. Equation (54) represents the CET aggregation function and essentially determines the price of aggregate exports PET.

$$\begin{cases} XW^s_{r,i,d} = \gamma^w_{r,i,d} XET_{r,i} \left(\dfrac{PE_{r,i,d}}{PET_{r,i}} \right)^{\omega^w_{r,i}} & \text{if } \omega^w_{r,i} \neq \infty \\ PE_{r,i,d} = PET_{r,i} & \text{if } \omega^w_{r,i} = \infty \end{cases} \tag{53}$$

$$PET_{r,i} XET_{r,i} = \sum_d PE_{r,i,d} XW^s_{r,i,d} \tag{54}$$

In the case of perfect transformation at both levels, the equilibrium condition can be replaced by the following expression, which is equivalent to the two market clearing conditions in the GEMPACK code (`MKTCLTRD_MARG` and `MKTCLTRD_NMRG`).

$$XS_{r,i} = XD^s_{r,i} + \sum_d XW^s_{r,i,d}$$

This subunit determines the variables XD^s, XET, XS, XW^s and PET corresponding to only the GEMPACK variable `qo` (output) as the other variables relate to the CET implementation.

39 The zero profit expression could be replaced by either the CET primal or dual expression.

3.13 International trade and transport margins

The GTAP database incorporates a wedge between the FOB and CIF price of goods, i.e. the border price of exports and imports. The wedge represents a trade and transport margin. These margins generate a demand for trade and transport services. The "global" trade and transport services sector purchases these services from various source regions using a CES production function. The first set of equations deals with the demand for international trade and transport services. Equation (55) determines the total demand for each of the transportation nodes, $XWMG$, with a simple Leontief assumption. The second equation, (56), breaks out total demand for international trade and transport services by node into different modes indexed by m.[40] The second level nest also uses a Leontief specification with the additional possibility of technical changes across modes (and nodes). The GEMPACK code has a single nest, i.e. the first two equations are collapsed into a single equation (`QTRANS_MFSD`). The GAMS code could eventually be extended to allow for substitution across modes as a function of relative prices. Equation (57) determines the aggregate price of international trade and transport services for each trade node. The expression relies on the price of each mode of transportation, which for lack of additional information, is assumed to be a global price and not specific to the trade node. Equation (58) determines the global demand for trade and transport services, $XTMG$, for each mode m.

$$XWMG_{r,i,d} = \zeta^{mg}_{r,i,d} XW^{d}_{r,i,d} \tag{55}$$

$$XMGM_{m,r,i,d} = \frac{\alpha^{mg}_{m,r,i,d}}{\lambda^{mg}_{m,r,i,d}} XWMG_{r,i,d} \tag{56}$$

$$PWMG_{r,i,d} = \sum_m \frac{\alpha^{mg}_{m,r,i,d}}{\lambda^{mg}_{m,r,i,d}} PTMG_m \tag{57}$$

$$XTMG_m = \sum_r \sum_i \sum_d XMGM_{m,r,i,d} \tag{58}$$

This subunit generates the variables $XWMG$, $XMGM$, $PWMG$ and $XTMG$, which correspond to the GEMPACK variables `qtmfsd`, `ptrans` and `qtm`. There is no corresponding GEMPACK variable for $XWMG$.

Variable $XTMG$ represents global demand for trade and transport services by mode m. There is a global supplier that purchases these services across the regions of the world minimizing cost and using a CES production function. Equation (59) represents the demand for international trade and transport services for mode m sourced in region r. The substitution elasticity across suppliers is given by the elasticity σ^{mg}.[41] Note that the GAMS code generates a demand at the Armington level that will eventually be allocated across domestic and imported goods. Since import shares are zero in the base data, the Armington variable will be equal to the domestic component, corresponding to the variable `qst` in the GEMPACK code. Equation (60) determines the average global supply price for each mode m.[42]

[40] In the full GTAP database, there are three modes of transportation—air, water and other transport.

[41] The GEMPACK code explicitly assumes a Cobb-Douglas specification, i.e. a substitution elasticity of 1.

[42] In the GEMPACK code, the component price is `pm` which corresponds to the GAMS output price *psychology*. The GAMS specification allows for two modifications. First, it assumes that the margin services are an Armington good. If instead the margin services were assumed to be explicitly a domestic good, the relevant component price would be PD. If, moreover, the GAMS code assumed perfect transformation of domestic output the relevant component price would be PS, the same as in the GEMPACK code.

$$XA_{r,m,tmg} = \alpha^a_{r,m,tmg} XTMG_m \left(\frac{PTMG_m}{PA_{r,m,tmg}}\right)^{\sigma^{mg}_{r,m}} \tag{59}$$

$$PTMG_m = \left[\sum_r \alpha^a_{r,m,tmg} PA^{(1-\sigma^{mg}_{r,m})}_{r,m,tmg}\right]^{1/(1-\sigma^{mg}_{r,m})} \tag{60}$$

This subunit determines the variables $XA_{r,m,tmg}$ and $PTMG_m$ corresponding to the GEMPACK variables `qst` and `pt`.

3.14 Bilateral trade prices

There are four bilateral trade prices corresponding to three price wedges. Producers in region r receive the price PE for commodity i delivered to region d. In the case of perfect transformation, the price PE is equal the aggregate supply price given by PS. Between the farm- or factory-gate, a bilateral export tax or subsidy (τ^e) is applied to the producer price and determines the export border price (or the free on board—FOB price), equation (61). The export price equation also incorporates an additional tax, ζ^e, which is uniform across destinations and allows for an exogenous and uniform shift in the export tax. It is initially set to 0. Equation (62) determines the import border price, PM^{cif}. Given the assumption of the Leontief demand for international trade and transport services, the import border price (or the cost, insurance and freight—CIF price) is equal to the FOB price augmented by the unit cost of the trade margin, equation (62). The final wedge represents the bilateral import tariffs (τ^m) that converts the CIF price of imports to the market bilateral price of imports, PM, equation (63).[43] The formulation allows for a uniform shift (across source countries) as represented by the tax variable ζ^m, which is initialized at 0.

$$PE^{fob}_{r,i,d} = PE_{r,i,d}(1 + \tau^e_{r,i,d} + \zeta^e_{r,i}) \tag{61}$$

$$PM^{cif}_{s,i,r} = PE^{fob}_{s,i,r} + \zeta^{mg}_{s,i,r} PWMG_{s,i,r} \tag{62}$$

$$PM_{s,i,r} = PM^{cif}_{s,i,r}(1 + \tau^m_{s,i,r} + \zeta^m_{r,i}) \tag{63}$$

This subunit generates PE^{fob}, PM^{cif} and PM corresponding to GEMPACK variables `pfob`, `pcif` and `pms`.

3.15 Market equilibrium

There are fundamentally two market equilibrium conditions for goods and services. The first guarantees equality of supply and demand for domestically produced goods sold on the domestic market. The second guarantees equality of supply and demand for each bilateral trade node. Equation (64) represents the equilibrium condition for the domestic market and essentially determines the equilibrium price PD. The supply side is determined from the CET domestic allocation specification. The demand side is determined by the top level Armington specification. Equation (65) reflects supply/demand equilibrium for each bilateral trade node, essentially determining the price PE. In the GAMS implementation, the latter equation is substituted out and the code only carries a variable XW without any superscripts.

[43] This is not the final price of imports because a sales tax is applied to the aggregate import volume.

$$XD^s_{r,i} = \sum_{aa} XD_{r,i,aa} \tag{64}$$

$$XW^s_{r,i,d} = XW^d_{r,i,d} \tag{65}$$

This subunit determines PD and PE. They have no equivalent in the GEMPACK code because they are linked to the CET specification of domestic production and are equal to the GAMS variable PS corresponding to the GEMPACK variable `pm`.[44] The equilibrium condition in the GEMPACK code is described above.

3.16 Factor markets

The GAMS version of factor market equilibrium allows for three possible specifications—two of which correspond to the GEMPACK specification. First, factors are either partially or fully mobile across sectors at the regional level and both of these are specified using a CET supply allocation mechanism with an allowance for perfect mobility. In GEMPACK, factors are either classified as mobile or sluggish, with the latter corresponding to a CET specification with a finite transformation elasticity. The GAMS version allows explicitly for a sector-specific resource, i.e. a resource that is perfectly immobile. This is treated differently from the CET with a zero transformation elasticity, because the sector specific resource is given an upward sloping supply curve. The set f of factors is therefore split into two subsets. One subset, fm, contains all mobile factors and will be governed by the CET specification with a transformation elasticity that can vary from 0 to ∞. The second, fnm, contains factors that are treated as sector specific. Their sector-specific supply will be specified using an upward sloping supply curve (eventually with zero elasticity). The standard GAMS version of the model follows the GEMPACK specification and all factors are classified as mobile. The natural sector-specific factors (essentially natural resources such as trees, primary energy reserves, etc.) are given a transformation elasticity that is very low (0.001).

Equation (66) determines the aggregate supply of mobile factors, XFT. There is no equivalent in the GEMPACK code where the aggregate supply of all factors is exogenous.[45] In this formulation, the supply curve is a function of the return to the aggregate factor, relative to an economy-wide price given by the variable $PABS$, which is a price index of domestic absorption and further described below. Setting the supply elasticity ($\eta^{ft}_{r,fm}$) to zero would have the same impact as exogenizing total supply. Equation (67) determines the factor supply to each sector under one of three market specifications. The first two lines relate to mobile factors only. The first is the standard CET supply function for partially mobile factors (e.g. land and natural resources in the case of the standard GTAP model). The second line holds for perfectly mobile factors (e.g. unskilled and skilled labor and capital in the standard GTAP model). With perfect mobility, the market price of each factor is uniform across all sectors. The third line holds only for sector-specific factors such as natural resources. In this case supply is specified as an upward sloping supply curve with the possibility of a zero supply elasticity. Equation (68) determines the price of the aggregate factor bundle for mobile factors. Equation (69) represents the factor supply equilibrium condition, i.e. supply equals demand at the level of each sector (i.e. production activity). For perfectly mobile factors, the equilibrium condition will actually be represented by equation (68) that equates the sum of demand to total supply. The equilibrium condition is substituted out of the model and the

[44] To reiterate once more, in the case of a diagonal 'make' matrix, the aggregate supply price PS is equal to the producer price (or market price) PP.
[45] TO BE CONFIRMED.

superscripts are eliminated from the variable XF. The final equation in this section, (70), links the equilibrium (or market) price of factors to the purchasers' (or agents') price of factors.

$$XFT_{r,fm} = A^{ft}_{r,fm} \left(\frac{PFT_{r,fm}}{PABS_r} \right)^{\eta^{ft}_{r,fm}} \tag{66}$$

$$\begin{cases} XF^s_{r,fm,a} = \gamma^f_{r,fm,a} XFT_{r,fm} \left(\frac{PF_{r,fm,a}}{PFT_{r,fm}} \right)^{\omega^f_{r,fm}} & \text{if } \omega^f_{r,fm} \neq \infty \\ PF_{r,fm,a} = PFT_{r,fm} & \text{if } \omega^f_{r,fm} = \infty \\ XF^s_{r,fnm,a} = \gamma^f_{r,fnm,a} \left(\frac{PF_{r,fnm,a}}{PABS_r} \right)^{\eta^{ff}_{r,fnm}} \end{cases} \tag{67}$$

$$\begin{cases} PFT_{r,fm} = \left[\sum_a \gamma^f_{r,fm,a} PF^{(1+\omega^f_{r,fm})}_{r,fm,a} \right]^{1/(1+\omega^f_{r,fm})} & \text{if } \omega^f_{r,fm} \neq \infty \\ XFT_{r,fm} = \sum_a XF^s_{r,fm,a} & \text{if } \omega^f_{r,fm} = \infty \end{cases} \tag{68}$$

$$XF^s_{r,f,a} = XF^d_{r,f,a} \tag{69}$$

$$PF^a_{r,f,a} = PF_{r,f,a} \left(1 + \tau^{ft}_{r,f,a} + \tau^{fs}_{r,f,a} \right) \tag{70}$$

This subunit generates XFT, XF^s, PFT, PF and PF^a. The corresponding GEMPACK variables are `qoes` (sectoral supply of sluggish factor), `pmes` (factor price of sluggish factor), `pm` (price of aggregate sluggish factor or equilibrium price of perfectly mobile factor) and `pfe` (tax-inclusive price of factors by sector).

3.17 Investment behavior

Regional savings is determined by the top level utility function. All savings are collected by a 'global' saver that then allocates savings across the regions of the model thereby determining regional investment. There are several specifications for the behavior of the global saver. In the first specification, global savings are allocated across regions so as to equalize 'risk' adjusted rates of return. In a second option, net new investment is allocated across regions using the same proportions as in the baseline.[46] A third option, oft-used in CGE models, fixes the capital account. These latter two options, imply that deviations of risk adjusted rates of return can occur.

Equation (71) determines the level of the beginning of period capital stock. The GAMS version of the model carries two versions of regional capital stocks. The first is the 'normalized' version that equals aggregate capital remuneration in the base period. This is the notion of the capital stock that is allocated across sectors and whose price is equal to 1 in the base period. The 'non-normalized' level corresponds to the initial estimate of the value of the beginning of period capital stock. Typically, the ratio of the normalized level to the non-normalized level represents the gross rate of return to capital in the base period. The non-normalized level should also represent a low multiple of aggregate GDP (say between 2 and 3 for most countries), and its value should be compatible, in terms of units, with the volume of investment. The initial value for K^0 comes from the header array `VKB` and the parameter χ^k is calibrated using base period data. The GEMPACK code does not require both variables since it is expressed in terms of percent change—and both

[46] TO BE CONFIRMED.

variables will have the same percentage change. Equation (72) calculates the end-of-period capital stock. It is equal to the depreciated level of the initial capital stock augmented by the in-period volume of investment (XI). Equation (72) is only valid with the non-normalized definition of the capital stock.

$$K_r^0 = \chi_r^k XFT_{r,cap} \tag{71}$$

$$K_r^1 = (1-\delta_r)K_r^0 + XI_r \tag{72}$$

Equation (73) defines the after-tax return to aggregate capital. The variable PFT represents aggregate capital remuneration to the normalized level of capital.[47] It is multiplied by χ^k to convert the return into return per the non-normalized level of the capital stock. Finally, it is adjusted by the tax rate on capital earnings to provide the net-of-tax return to the aggregate owner of capital. Equation (74) defines the net return to capital after adjusting for the replacement cost of capital (similar to Tobin's q) and the rate of depreciation. Equation (75) defines the expected rate of return, R^e. It is equal to the contemporaneous net rate of return, R^c, adjusted by a factor that depends on future increases to the capital stock. As the capital stock increases, all else equal, one expects the return to decline. The level of adjustment depends on the elasticity ϵ^{RoR}.[48]

$$R_r^a = \chi_r^k PFT_{r,cap}(1-\kappa_{r,cap}^f) \tag{73}$$

$$R_r^c = \frac{R_r^a}{PI_r} - \delta_r \tag{74}$$

$$R_r^e = R_r^c \left(\frac{K_r^1}{K_r^0}\right)^{-\epsilon_r^{RoR}} \tag{75}$$

This subunit defines K^0, K^1, R^a, R^c and R^e corresponding to the GEMPACK variables `kb`, `ksvces`, `rental`, `rorc` and `rore`.

There are three possible closure rules for investment.[49] Equation (76) reflects reflects the specific closure for each of the three options, which is determined by the user-specified flag `RoRFlag`. With `RoRFlag` set to 1, closure is defined by the equality of the expected risk adjusted rates of return to the global rate of return, where π_r^r is the risk adjustment and calibrated with base year information.[50] With `RoRFlag` set to 0, closure reflects that net investment growth across all regions is equal to the global growth in investment, or in other words, the global allocation of net investment reflects the initial allocation of net investment, where the parameter χ^I is calibrated to the initial volumes and is fixed. Note, that for consistency purposes, this form of the equation is defined over $n-1$ regions. The final option, with `RoRFlag` set to 2, fixes the capital account for each region as defined by the variable S^f. Again, the capital flows are fixed for only $n-1$ of the regions and the capital account consistency equation will determine the capital flow for the residual region. Equation (77) defines the global rate of return, R^g for the cases where the net investment ratio or the capital account is fixed. It is defined as the weighted sum of the regional expected rates of return and the weights are provided by the regional shares of (net) investment in global (net) investment, equation (78).

[47] The variable PFT is defined over all factors with the index *cap* referring to the price of capital.

[48] The elasticity ϵ^{RoR} corresponds to the `RORFLEX` parameter in the GEMPACK code. See Hertel [1997] for further elaboration.

[49] In the standard GTAP model, there are two potential closures.

[50] The GEMPACK version does not require the risk adjustment as in the percentage change formulation the risk adjustment factor drops out if it is exogenous.

It is purely definitional and does not interact with the rest of the model. Equation (79) reflects the capital flow consistency requirement, i.e. the sum of net capital flows across all regions must be 0. With the expected rate of return closure, this equation in essence determines the global rate of return, R^g. With the constant investment ratio closure, this equation determines the volume of investment for the residual region. For the fixed capital account closure, this equation determines the capital flow to the residual region, all other net capital flows are fixed. Equation (80) determines the level of gross nominal investment. It is equal to the value of depreciation (at replacement cost), regional savings, and net foreign capital flows. The latter are evaluated at the average global cost of investment goods, PI_{GBL}. Note, that due to Walras' Law, the equation is only specified for $n-1$ regions.

$$\begin{cases} \pi_r^r R_r^e = R^g & \text{if } \texttt{RoRFlag} = 1 \\ XI_r - \delta_r K_r^0 = \chi_r^I XI_{GBL} & \text{if } \texttt{RoRFlag} = 0 \\ S_r^f = \overline{S_r^f} & \text{if } \texttt{RoRFlag} = 2 \end{cases} \tag{76}$$

$$R^g = \sum_r \varphi_r^r R_r^e \quad \text{if} \quad \texttt{RoRFlag} \neq 1 \tag{77}$$

$$\varphi_r^r = \frac{PI_r \left(XI_r - \delta_r K_r^0\right)}{\sum\limits_s PI_s \left(XI_s - \delta_s K_s^0\right)} \tag{78}$$

$$\sum_r S_r^f \equiv 0 \tag{79}$$

$$YI_r = \delta_r PI_r K_r^0 + Save_r + PI_{GBL}.S_r^f \tag{80}$$

Equation (81) defines global (net) investment, XI_{GBL}, with equation (82) defining the average global price of purchasing investment goods, PI_{GBL}. The latter is used to value net foreign savings. The savings deflator, used to evaluate the utility from savings, is defined in equation (83).

$$XI_{GBL} = \sum_r \left[XI_r - \delta_r K_r^0\right] \tag{81}$$

$$PI_{GBL} XI_{GBL} = \sum_r PI_r \left(XI_r - \delta_r K_r^0\right) \tag{82}$$

$$PSave_r = PI_r + \sum_s \frac{PI_{GBL} S_s^f}{PI_{GBL} XI_{GBL}} PI_s = PI_r + \frac{\sum_s S_s^f PI_s}{XI_{GBL}} \tag{83}$$

This subunit generates S^f, R^g, φ^r, YI, XI_{GBL}, XI_{GBL} and $PSave$. These correspond to `rorg`, `qcgds`, `globalcgds`, `pcgdswld` and `psave`.[51]

3.18 Price indices and model numéraire

The equations below define four price indices—two of which are regional and two of which are global. Equation (84) defines a price index for regional absorption. It is a weighted average of the Armington prices covering private, public and investment demand (as captured by the subset *fd* of *aa*.) Equation (85) defines a price index of so-called manufactured exports from high-income

[51] TO BE VERIFIED, and what corresponds to S^f in the GEMPACK code?

countries—intended to be closely related to the World Bank's Manufactured Unit Value (MUV) index. The first regional sum covers all regions defined in the subset HIC. The commodity index covers all commodities defined in the $MANU$ index. The second regional sum is over all regions. The index is a weighted average of the export prices evaluated at the border, i.e. at FOB prices. Equation (86) defines a second regional price index and it is an index of factor prices (evaluated at market prices). Equation (87) is a similar index, but is an index of global factor prices. Equation (88) defines the global model numéraire. The GEMPACK default is the global factor price index, but any global price index could replace $PFACT^w$, such as $PMUV$. The price itself is exogenous, but the model is nonetheless square as one of the investment equations is dropped.

$$PABS_r = \sum_i \sum_{fd} \varphi^a_{r,i,fd} PA_{r,i,fd} \tag{84}$$

$$PMUV = \sum_{r \in HIC} \sum_{i \in MANU} \sum_s \varphi^w_{r,i,s} PE^{fob}_{r,i,s} \tag{85}$$

$$PFACT_r = \sum_f \varphi^f_{r,f} PFT_{r,f} \tag{86}$$

$$PFACT^w = \sum_r \sum_f \varphi^w_{r,f} PFT_{r,f} \tag{87}$$

$$PNUM = PFACT^w \tag{88}$$

This subunit generates $PABS$, $PMUV$, $PFACT$ and $PFACT^w$. Equation (88) defining the numéraire in essence determines investment for the residual region that is left out of the model specification. The corresponding variables in the GEMPACK version are `pfactor` and `pfactwld` for the two factor price indices—the others having no correspondence in the GEMPACK version.

3.19 Technology

TO BE DONE

3.20 Summary of price relations

This section summarizes the multiple relations across all prices. For the goods markets there are two sets of equilibrium prices from which all other prices can be derived. These are the markets for goods produced domestically and sold on the domestic market, PD, and the bilateral export prices, PE. Figure 5 illustrates the downstream linkages between the price of domestic output and the subsequent market prices. In the presence of constant-returns-to-scale technology and perfect competition, the unit cost of production, PX, also represents the pre-tax producer price. The market price of output, PP, takes into account the production tax, τ^p. Domestic output, XP, is converted to domestic supply, XS, using the 'make' matrix, with a corresponding transformation of the output price PP to the price of aggregate domestic supply PS. Domestic supply is sold to both domestic and export markets. The price on the former is PD. The producer receives price PE from each destination country—though due to the nested nature of the allocation of aggregate output, there is an intermediate aggregate price of exports represented by PET. Between the farm- or factory-gate, there is an additional tax or subsidy on exports, τ^e, that generates the border price of exports, PE^{fob}. From the source border to the destination border, transportation and trade margins are tacked on, as represented by the margin wedge, ζ^{mg}, leading to the border price of imports, PM^{cif}. The bilateral market of imports, PM, is equal to the border price adjusted by import taxes.

	PX			Unit cost of production
	$\Downarrow$		τ^p	Output tax
	PP			Market price of domestic production
	$\Downarrow$			'Make' matrix transformation of domestic production
	PS			Market price of total domestic supply
$\swarrow$		$\searrow$		Top level CET transformation (ω^x)
PD		PET		Market price of domestic & export goods
		$\Downarrow$		Second level CET transformation (ω^w)
		PE_d		(Bilateral) market price of export goods
		$\Downarrow$	τ^e, ζ^e	Taxes on exports
		PE_d^{fob}		Border price of exports (FOB)
		$\Downarrow$	ζ^{mg}	International trade & transport margins
		PM_d^{cif}		Border price of imports (CIF)
		$\Downarrow$	τ^m, ζ^m	Taxes on imports
		PM_d		(Bilateral) market price of imports

Figure 5: **Summary of prices from the supply side**

The summary of prices from the demand side will pick up from the border price of imports, PM^{cif}, and the market price of domestic production sold on the domestic market, PD, figure 6. The bilateral market price of imports, PM, is equal to the border price adjusted by import taxes. Imports are aggregated across source regions to form an aggregate import composite good with a price given by PMT. Agent-specific sales tax are then applied to domestic goods and the aggregate import good to form their end-user prices, respectively PD^p and PM^p. These last prices are then used to form domestic absorption, XA, with a price of PA that represents the aggregate, tax-inclusive price of domestic and imported goods.

		PM_s^{cif}		Border price of imports (CIF)
		$\Downarrow$	τ^m, ζ^m	Taxes on imports
		PM_s		(Bilateral) market price of imports
		$\Downarrow$		Second level Armington CES (σ^w)
PD		PMT		Market price of domestic & (aggregate) import goods
$\Downarrow$		$\Downarrow$	$\tau^{dtx}, \tau^{mtx}, \chi^{tx}$	Sales tax on domestic & imported goods
PD^p		PM^p		End-user price of domestic & (aggregate) import goods
	$\searrow$	$\swarrow$		Top level CES aggregation of domestic & import goods (σ^m)
	PA			Armington price of goods (by agent)

Figure 6: **Summary of prices from the demand side**

The remaining key price wedge in the model regards factor prices. The equilibrium (or market) price for factors is given by PF. The agents' price of factors (or the cost of factors to producers), is given by PF^a, which is the market price adjusted by factor taxes and subsidies as represented by τ^{ft} and τ^{fs}, respectively.

4 Accounting framework

This section describes the accounting framework used by the GAMS version of the model. Two different frameworks will be described. The first describes the analytical SAM underlying the model, i.e. how the SAM can be constructed using the variables of the model. The second describes the links between the model variables and the initial database as provided by GTAP. The SAM is not a standard SAM and does not represent the full functionality of the model nor the underlying database. For example, demand is specified at the Armington level and the sales tax are collapsed to a single row dimension.

4.1 The analytical SAM

The analytical SAM is the accounting framework that is derived from the variables of the model. In the absence of a shock, the analytical SAM should reproduce the input SAM. Table 2 re-produces the analytical SAM. Below are a few explanatory notes:

1. The regional index (r) is dropped for most of the expressions except when needed, for example in the bilateral trade flows.

2. The SAM departs from standard accounting principles in a number of ways. First, the law of one price does not hold across all rows. For example, the activities' rows contains the value of Armington consumption at pre-tax prices. The market prices of domestic goods, PD, and aggregate imported goods, PMT, are uniform across all agents, but not the Armington price.

3. For the sake of exposition, the final demand columns have been collapsed into a single column labeled `FDM` that covers the final demand index fd. Taken literally, this implies that the analytical SAM is not square as the revenue rows for the final demand agents are fully specified. However, all of the entries of the final demand expenditure accounts have the same generic formula.

4. The SAM reflects the differentiation between activities and commodities. The intersection of the activity rows with the commodity columns represents the transformation of domestic production into commodities. In the standard version of the model, the matrix is diagonal and there is a one-to-one mapping between activities and commodities.

5. The regional household is explicit in this version of the SAM. It could be dropped in which case there would need to be balancing items across the household, government and investment accounts.

6. The trade rows under the commodity columns represents the bilateral imports for region r. These are priced at border, i.e. CIF, prices. The commodity rows under the trade columns represent bilateral exports from region r and are valued at border, i.e. FOB, prices. At the global level, the difference between the two matrices represents the aggregate value of the trade and transport margins.

7. The `BOP` row under the trade columns represents aggregate imports from each region (at border prices). Similarly, the trade rows under the `BOP` column represents the aggregate

exports of region r towards each region. The BOP accounts reflect the full balance of payments. For each region, the sum of exports (including the export of trade and transport services) less the sum of imports equals the net flow of foreign savings. At the global level, the foreign savings cell should be zero.

4.2 Correspondence to GTAP database

Table 3 provides the correspondence between the variables in the GAMS model and the standard GTAP database that is used as the starting basis for the GTAP model. Below are a few notes on the table:

1. Subsidy costs in the GTAP database are entered as positive values. In the GAMS version of the model, subsidy rates are treated as negative tax rates and hence the costs will be negative. This affects the variables `OSEP` and `FBEP`.

2. The GTAP variables `VIFM`, `VDFM`, `VIFA` and `VDFA` contain the investment column from the GAMS version of the model. In GTAP, this is labeled as `CGDS`. It is stored in a separate variable in the GAMS version.

3. The GTAP database assumes a one-to-one correspondence between activities and commodities, which would be reflected into a one-to-one mapping between index i for commodities and index a for activities. The aggregation facility creates the diagonal 'make' matrix at the fully disaggregated level and uses separate mapping definitions for activities and commodities. This provides the user with full flexibility to create activities with joint production and commodities produced by more than one activity.

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Table 2: Analytical SAM

	$\boldsymbol{ACT}(a)$	$\boldsymbol{COMM}(i)$	$\boldsymbol{FCT}(f)$
Activities(a)		$P_{a,i}X_{a,i}$	
Commodities(i)	$PD_iXD_{i,a} + PMT_iXM_{i,a}$		
Factors(f)	$PF_{f,a}XF_{f,a}$		
Indirect tax	$\sum_i \left[\tau^d_{i,a}PD_iXD_{i,a} + \tau^m_{i,a}PMT_iXM_{i,a}\right]$		
Production tax	$\tau^p_a PX_a XP_a$		
Factor tax	$\sum_f \tau^{ft}_{f,a}PF_{f,a}XF_{f,a}$		
Factor subsidy	$\sum_f \tau^{fs}_{f,a}PF_{f,a}XF_{f,a}$		
Import tax		$\sum_s \tau^m_{s,i,r}PM^{cif}_{s,i,r}XW_{s,i,r}$	
Export tax		$\sum_d \tau^e_{r,i,d}PE_{r,i,d}XW_{r,i,d}$	
Direct tax			$\kappa^f_f PFT_f XFT_f$
Regional household			$(1-\kappa^f_f)PFT_f XFT_f - \delta.PI.K^0$
Private			
Government			
Investment			$\delta.PI.K^0$
Trade margins			
Trade(s)		$PM^{cif}_{s,i,r}XW_{s,i,r}$	
Balance of payments			

Table 2 Analytical SAM, ctd.

	ITX	***PTX***	***FTX***	***FTS***	***MTX***	***ETX***	***DTX***	***RHH***
Activities(a)								
Commodities(i)								
Factors(f)								
Indirect tax								
Production tax								
Factor tax								
Factor subsidy								
Import tax								
Export tax								
Direct tax								
Regional household	$YTAX_{at}$	$YTAX_{pt}$	$YTAX_{ft}$	$YTAX_{fs}$	$YTAX_{mt}$	$YTAX_{et}$	$YTAX_{dt}$	
Private								YC
Government								YG
Investment								*Save*
Trade margins								
Trade(s)								
Balance of payments								

Table 2 Analytical SAM, ctd.

	$\boldsymbol{FDM}(fd)$	$\boldsymbol{TRD}(d)$	$\boldsymbol{BOP}$
Activities(a)			
Commodities(i)	$PD_i XD_{i,fd} + PMT_i XM_{i,fd}$	$PE^{fob}_{r,i,d} XW_{r,i,d}$	
Factors(f)			
Indirect tax	$\sum_i \left[\tau^d_{i,fd} PD_i XD_{i,fd} + \tau^m_{i,fd} PMT_i XM_{i,fd}\right]$		
Production tax			
Factor tax			
Factor subsidy			
Import tax			
Export tax			
Direct tax			
Regional household			
Private			
Government			
Investment			$PI_{GBL} S^f$
Trade margins			$\sum_m PA_{m,tmg} XA_{m,tmg}$
Trade(s)			$\sum_i PE^{fob}_{r,i,s} XW_{r,j,s}$
Balance of payments		$\sum_i PM^{cif}_{d,i,r} XW_{d,j,r}$	

Table 3: Correspondence between GAMS model variables and GTAP database

GTAP	***GAMS***	***Description***
VIFM	$PMT_{r,i}XM_{r,i,a}$	Firms' import purchases at market prices
VDFM	$PD_{r,i}XD_{r,i,a}$	Firms' domestic purchases at market prices
VIFA	$PM^{a}_{r,i,a}XM_{r,i,a}$	Firms' import purchases at agents' prices
VDFA	$PD^{a}_{r,i,a}XD_{r,i,a}$	Firms' domestic purchases at agents' prices
VIPM	$PMT_{r,i}XM_{r,i,h}$	Private import purchases at market prices
VDPM	$PD_{r,i}XD_{r,i,h}$	Private domestic purchases at market prices
VIPA	$PM^{a}_{r,i,h}XM_{r,i,h}$	Private import purchases at agents' prices
VDPA	$PD^{a}_{r,i,h}XD_{r,i,h}$	Private domestic purchases at agents' prices)
VIGM	$PMT_{r,i}XM_{r,i,gov}$	Government import purchases at market prices
VDGM	$PD_{r,i}XD_{r,i,gov}$	Government domestic purchases at market prices
VIGA	$PM^{a}_{r,i,gov}XM_{r,i,gov}$	Government import purchases at agents' prices
VDGA	$PD^{a}_{r,i,gov}XD_{r,i,gov}$	Government domestic purchases at agents' prices)
VIFM(CGDS)	$PMT_{r,i}XM_{r,i,inv}$	Investment import purchases at market prices
VDFM(CGDS)	$PD_{r,i}XD_{r,i,inv}$	Investment domestic purchases at market prices
VIFA(CGDS)	$PM^{a}_{r,i,inv}XM_{r,i,inv}$	Investment import purchases at agents' prices
VDFA(CGDS)	$PD^{a}_{r,i,inv}XD_{r,i,inv}$	Investment domestic purchases at agents' prices
VFM	$PF_{r,f,a}XF_{r,f,a}$	Firms' factor purchases at market prices
EVFA	$PF^{a}_{r,f,a}XF_{r,f,a}$	Firms' factor purchases at agents' prices
FBEP	$-[\tau^{fs}_{r,f,a}PF_{r,f,a}XF_{r,f,a}]$	Value of factor demand subsidies
FTRV	$\tau^{ft}_{r,f,a}PF_{r,f,a}XF_{r,f,a}$	Value of factor demand taxes
TVOM	$PP_{r,a}XP_{r,a}$	Value of production at market (producer prices)
OSEP	$-[\tau^{p}_{r,a}PX_{r,a}XP_{r,a}]$	Value of output taxes/subsidies
EVOA	$(1-\kappa^{f}_{r,f})PFT_{r,f}XFT_{r,f}$	After tax factor remuneration
VKB	K^{0}_{r}	Beginning of period capital stock
VDEP	$\delta_{r}PI_{r}K^{0}_{r}$	Value of depreciation
SAVE	$Save_{r}$	Domestic savings
VIMS	$PM_{s,i,r}XW_{s,i,r}$	Value of bilateral imports tariff inclusive
VIWS	$PM^{cif}_{s,i,r}XW_{s,i,r}$	Value of bilateral imports at border prices
VXWD	$PE^{fob}_{r,i,d}XW_{r,i,d}$	Value of bilateral exports at border prices
VXMD	$PE_{r,i,d}XW_{r,i,d}$	Value of bilateral exports at producer prices
VST	$PD_{r,m}XD_{r,m,tmg}$	Domestic exports of trade & transport services
VTWR	$PTMG_{m}XMGM_{m,r,i,d}$	Value of margins from r to d for good i using mode m
POP	Pop_{r}	Population—millions

Thomas F. Rutherford. Gtap7ingams. Mimeo, Center for Energy Policy and Economics, ETH Zurich, 2010. URL `http://www.mpsge.org/GTAP8inGAMS.zip`.