

# **Introducing Firm Heterogeneity Theory into CGEBox**

Wolfgang Britz and Yaghoob Jafari  
Institute for Food and Resource Economics, University of Bonn, Germany

## **Abstract**

Computable general equilibrium models with firm productivity heterogeneity can add to the explanatory power of traditional computable general equilibrium by taking into account changes in average productivity and love of varieties. For this purpose, this paper introduces the Melitz (2003) model of heterogeneous firms into the CGEBox and discusses how to calibrate the model against the GTAP database. The implementation follows to a large extent from that of Balistreri et al (2013), and Zeynep et al. 2016 for the GTAP-HET model. The model can be flexibly combined with different features of CGEBox and applied to many sectors and detailed data bases. The numerical application of 50% reduction of all imports tariff and export subsidies with a 10x10 aggregation of the GTAP8 data base is illustrated, focusing on welfare changes and highlighting the newly available information from the model.

**Keywords:** Computable general equilibrium model, productivity heterogeneous firms, trade policy.

**Jel:** C68, F12,

## 1. Introduction

Since Armington (1969) proposed to treat imported and domestic varieties of goods in the same classification as imperfect substitutes depicted by a CES-utility function, this approach dominated applied Computable General Equilibrium (CGE) analysis. It provides a powerful, yet relatively simple framework for studying international trade policy, not at least as it can accommodate any observed pattern of trade flows and related prices, i.e. the intensive margin of trade. However, preferences for each origin in the Armington model are fixed such that changes in trade cannot impact average imported qualities on a trade link. It hence neglects potential variations at the extensive margin of trade such as trade flows in new products which are found as important in empirical analysis (Hummel and Klenow, 2005; Chenny, 2008, among others).

The pioneer paper by Melitz (2003) introduced firm productivity heterogeneity drawing from Hopenhayn (1992) into the monopolistic competition framework by Krugman (1980). The Melitz model combines changes at the intensive and extensive margins of trade by allowing firms to self-select new export markets based on their productivity level. Subsequently, many papers applying the model (Bernard et al., 2003, 2006, 2007; Eaton et al., 2004) supported its empirical evidence by reproducing salient trade patterns observed in recent micro level studies. As the Melitz model provides a more general framework to depict bi-lateral trade which has proven as empirically superior, there have been a number of efforts to introduce firm heterogeneity into CGE models (Zhai 2008; Baliatrerri et al., 2011; Oyamada, 2013; Akgul et al., 2016; and Dixon et al., 2016).

The first paper which introduced firm heterogeneity following Melitz (2003) into a CGE framework is Zhai (2008). The Zhai-Melitz approach captures variations in the extensive margin of trade flows in contrast to traditional CGE models based on Armington's (1969) assumption. Zhai (2008) provided the theoretically well-grounded empirical model based on the assumption of no free entry and exit. However, the Zhai implementation allows for adjustments in the extensive margin of trade only as a result of changed export share of firms engaging in a specific trade link, while limiting the variety gains brought by new entrants. Consequently, this assumption results in overestimation of the extensive margin of trade and in turn productivity effect. Baliatrerri et al. (2011) overcomes that restriction by allowing also for new entrants on each trade link and accounts for a certain exogenous share of firms leaving in each period the industry. Akgul, Vitoria and Hertel (2016) introduced the Melitz framework into the standard GTAP model, abstracting from exogenous firm exit.

Besides, Baliatrerri and Rutherford (2013) set out stylized versions of the Armington, Krugman, and Melitz under a unified treatment, and then compare the outcome of different approaches. Inspired by Baliatrerri and Rutherford (2013), Dixon et al. (2016) draws the connections between the three models by developing them sequentially as special cases of an encompassing model. Dixon et al. (2016) interestingly show that the Armington, Krugman, and Melitz models are progressively less restrictive special cases of a more generalized model, derived from a cost

minimization problem of a worldwide planner. While Dixon et al. (2016) provide an illustrative numerical general equilibrium model with Melitz sectors, using the earlier version of Dixon et al. (2016), Oyamada (2014) develops a simple AGE framework within which user can switch between Armington, Krugman, and Melitz models, and shows how the Dixon et al. (2016) framework can be parameterized<sup>1</sup>.

We discuss in here the introduction of the Melitz model into the CEGBox CGE modeling platform developed by van der Mensbrugghe and Britz (2015, hereafter M-B). Besides realized in GAMS (GAMS Development Corporation, 2013) and not in GEMPACK (Harrison and Pearson, 1996), it differs from the standard GTAP model (Hertel, 1997) in several aspects. Inter alia, its equations are all written in levels, whereas the GEMACK realization uses mostly equations depicting relative changes in linearized form. Furthermore, it aims at a modular and easily extendable framework. The implementation of the Melitz model discussed in here provides an example of such a modular extension.

The remainder of this paper is structured as follows: the next section (2) presents a brief review of the trade theories literature; section (3) describe the implementation of firm heterogeneity in the CEGBox model; section (4) presents the technical implementation of the CEGBox model with heterogeneous firms along with results of an example application; and finally section (5) concludes.

## 2. Brief Literature overview

With regard to international trade, traditional applied general equilibrium models fail in two important aspects (see critique by Keohe 2005): they do neither allow (trade) policies to impact on aggregate productivity nor on trade along the extensive margin. In order to overcome these shortcomings, a new type of international trade models has emerged drawing on the Melitz (2003) model. It considers heterogeneous firms under monopolistic competition which can self-select into export market.

In traditional trade theories, countries specialize in production and export of those commodities in which they have comparative advantages, in the Ricardian framework based on differences in technology and in the Heckscher-Ohlin one in endowments. These models assume perfect competition and constant returns to scale, often implying that firm size is indeterminate. Due to the resulting specialization, countries either import or export a certain product. Krugman (1980), Helpman (1981) and Ethier (1982) established the so-called “*new*” trade theories drawing on variety-based models. Here, firms specialize in distinct horizontally-differentiated varieties of a product which provides an explanation why countries simultaneously export and import within the same broader industry. From the demand side, that provides a specific interpretation of the

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<sup>1</sup> See also Itakura and Oyamada (2013) for technical aspects and Roson and Oyamada (2014) for a review of this emerging field.

Armington assumption where quality differences reflect these distinct varieties offered by firms. A key simplification in this strand of literature is the assumption of a representative firm within each country. It is also used by Helpman and Krugman (1985) who combine the traditional and new trade theory within an integrated equilibrium approach to provide an explanation for the pattern of trade where both inter-industry trade and “*cross-hauling*” can take place.

Increased availability of micro data on plant and firm level since the late 1980s generated empirically evidence challenging these theories of international trade. It became clear that firms are heterogeneous in terms of productivity, export behavior response to trade shocks, and other economic characteristics even within narrowly-defined industries (“one variety”). Specifically, analysis of micro datasets on firms and plants showed that *i*) only a small minority of firms actually engage in export (Bernard and Jensen 1995, Bernard et al. 2007) while there is considerable variation in export market participation rates across industries (Bernard et al. 2007); *ii*) exporters are more productive and larger than non-exporters (Bernard and Jensen 1995, Bernard et al. 2003,2007); *iii*) productivity dispersion exists across coexisting firms within any sector (Bartelsman and Dom 2000) and among exporters in the number of markets (Eaton et.al 2004); *iv*) there are substantial reallocation effects within an industry following trade liberalization episodes where more productive firm replaces less productive ones (Foster et.al 2001, Aw et al. 2001) increasing aggregate productivity; and *v*) and endogenous changes in firm productivity via shifting market shares influences within-industry resource allocation (Bernard et al. 2006,2010) and therefore the aggregate productivity.

None of the above-mentioned observations found in micro datasets can be explained with the simplifying assumption of a representative firm within countries or industries made both in the traditional and “*new*” trade theories. This led to the development of international trade models with firm heterogeneity like that of Melitz (2003). He introduced firm heterogeneity into the model of Krugman (1980) which considers horizontal differentiation into varieties and increasing returns to scale.

Further, the role of the extensive margin, i.e. export of goods not exported before, in observed international trade patterns recently received increased attention. Several studies highlighted the importance of new varieties in export markets and the related welfare implications (see Romer, 1994; Feenstra, 1994; Broda and Weinstein, 2006; Balistreri et al, 2010; Keohe and Ruhl, 2013; among others). That strand of literature is complemented by more theoretical works which discuss impacts of trade liberalization on the extensive margin (Hummel and Klenow, 2005; Chenny, 2008, among others). Traditional applied general-equilibrium models based on the Armington assumption cannot depict changes at the extensive margin of trade<sup>2</sup> as an important

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<sup>2</sup> Eaton et al. (2004) and Hillberry and Hummels (2008) defines the role of extensive margin in terms of firms serving a market while Hummels and Klenow (2005) and Broda and Weinstein (2006) identify the extensive margin in terms of the role of change in the number of products a firm trades or in the number of its trade partners (countries).

source of new trade (Kehoe, 2005). Heterogeneous firm based models like that of Melitz (2003) overcome that shortcoming by depicting changes simultaneously at the intensive and extensive margins which motivates the implementation into the CGEBox model discussed in here. The logic of the Melitz model is that actions that facilitate trade will raise both export variety and average productivity.

### **3. Implementation of Melitz model into the CGEBox modeling platform**

The standard GTAP model (Hertel 1997), developed by the Centre for Global Trade Analysis is a global, multi-regional, comparative static CGE based on neoclassical assumptions and equilibrium conditions that follow Walras' law. Policies are depicted by fixed relative price wedges. The model and variants thereof are the most widely used tools for the ex-ante analysis of economy-wide trade effects of multilateral or bilateral trade agreements. Based on a modified Cobb-Douglas Utility function, national income in each region is allocated among three types of final aggregate demand agents, namely government, private households, and savings. Each aggregate agent features its own Armington composite of domestic produce and aggregate imports for each product category, while the aggregate import composition for each product category is determined by a shared second Armington nest which also encompasses intermediate demand. Final demand expenditures on the aggregated Armington commodities reflect utility maximization, in the case of the representative private household drawing on a non-homothetic constant difference of elasticity expenditure (CDE) function; in case of the government and investment agents based on constant elasticity of substitution (CES) utility function. Markets are assumed competitive.

Production in each country and all sectors assumes a constant return to scale technology drawing on nested CES functions. In the standard GTAP model, the top level nest is a Leontief aggregate of value added and intermediate input use; the composition of the latter is based on fixed physical input coefficients. The value added nest allows for substitution between primary factors. As for the final demand agents, each sector features its own Armington nest to determine the composition of intermediate input demand for each commodity from domestic product and imports. The import composition is however identical across sectors and final demand, as mentioned above. Primary factors can either be assumed to be perfectly mobile across sectors such that the law of one price holds, or can be treated as “sluggish” based on Constant elasticity of transformation (CET) specification such that return to factors can differ between sectors.

In the standard GTAP framework, saving and capital is determined endogenously through a fictitious Global Bank. The Global Bank allocates investment across regions such that it equates the changes in the expected returns across countries. In the model, ad-valorem wedges can depict policy induces impacts on product price at the level of production, export, import and final consumption. The FOB (free on board) prices are differentiated by exporter and hence reflect bilateral export taxes or subsidies, adding international transport margins defines the CIF (cost,

insurance and freight price) for each importer to which import taxes or subsidies are added. That allows for a rather detailed analysis of trade policy.

We now turn to the implementation of the Melitz (2003) model into the CGEBox version developed by Van der Mensbrugge and Britz (2015). The model's equations are written in levels, and not as a mix of equations in levels and in linearized relative differences as found in GEMPACK based CGE implementations. That CGEBox<sup>3</sup> version allows for an exact replica of the standard GEMPACK version as developed by Hertel (1997). However, based on a flexible and modular code structure, it can also accommodate different assumptions of which we mention only some important ones. On the production side, non-diagonal make matrices, potentially combined with a CET approach are possible, while a flexible nesting approach allows more complex CES nests such as e.g. found in GTAP-AGR and GTAP-E. The Armington nests can be combined with a two-stage CET approach which allows for price dependent supply changes in with regard to the shares of domestic sales and total exports, respectively bi-lateral exports. The model can also be used in recursive-dynamic fashion. In the following, we only refer to the standard GTAP replica.

The actual implementation of the Melitz model into CGEBox draws to a large extent on the empirical method by Balistreri and Rutherford (hereafter BR) (2013) to introduce the Melitz (2003) model into an applied equilibrium model. Differences are detailed below. Further, we show how the Melitz structure compares to the Armington one. In our equation structure, sectors either are based on the standard representation, i.e. a two stage Armington structure on the demand side combined with constant return to scale industries, or follow the Melitz model with monopolistic composition and a different demand representation as detailed next.

The monopolistic competition sectors draw strictly on the framework of Melitz (2003): each firm produces one single unique variety over a continuum of varieties under conditions of monopolistic competition arising from imperfect substitution in demand for these varieties. Accordingly, the number of varieties produced in a regional industry is equal to the number of firms operating. New firms can freely enter an industry by paying a fixed entry cost which is thereafter sunk. However, before a firm enters, it is uncertain about its productivity level which becomes known once the sunk entry cost is paid. That productivity level is determined by a draw from a given productivity distribution. Once a firm knows its productivity level, it will only operate on those trade links, i.e. serving the domestic market or a specific export destination, where its profits are positive. The latter are defined as totals sales revenues on a trade link minus the bilateral fixed cost of trade plus per unit variable cost times sales quantity, the per unit variable cost are assumed to be independent from the output level and the trade link. All firms

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<sup>3</sup> Tom Rutherford provides since a long time a GAMS based implementation of a CGE called GTAPinGAMS which however differs somewhat to the standard GTAP model. The M-B implementation is coded in Dervis et al. (1982) tradition and departs significantly from the nomenclature used by Rutherford and that used by in the GEMPACK version of the model.

face the same bilateral fixed cost on each link; however these costs differ across the trade links. Accordingly, the individual firms' decision to operate on a specific link depends on its productivity level which determines per unit variable cost. At given fixed costs of bi-lateral trade, given potential demand for an additional variety on a trade link and the price received for it, there is hence a cut-off productivity ( $\tilde{\varphi}_{rs}$ ) level beyond which profits become negative.

Those firms which draw a productivity level higher than that zero-profit cutoff productivity will operate on the trade link and those below the cutoff level will not. The firm with the productivity level equal to the cut off level is called the marginal firm and faces zero profits on that link; all other firms operating on that link make positive ones. A reduction in bi-lateral fixed cost of trade or higher demand, for instance from trade liberalization, opens hence a window of opportunity for less productive firms to serve new trade links which benefits consumer by providing more diversity in the product bundle imported from a specific origin.

There is no restriction on the number of markets that each firm can serve. The same firm is typically not active on all trade links as they differ in fixed bilateral cost of trade. Since serving the domestic markets requires lower bilateral (here country to same country) fixed cost, the more productive firms participate in export markets. Considering that intra-industry differentiation allows to depict impacts of trade policy changes on captured in conventional trade models where a trade policy leads only to re-allocation of resources between industries. In firm heterogeneity models however, a trade policy induces additionally re-allocation of resources within each industry. Firms can expand their market shares by absorbing resources of less productive ones forced to exit.

In these models, a policy that reduces the worldwide barriers to trade thus increases profits that existing exporters can earn in foreign markets and reduces the export productivity cutoff above which firms export. Input demand within the industry rises, due both to expansion by existing exporters and to new firms beginning to export. The increase in input demand bids up factor prices and reduces the profits of non-exporters. Reduction in profits in the domestic market induces some low productivity firms who were previously marginal to exit the industry. As low productivity firms exit and output and production factors are reallocated towards higher-productivity firms, average industry productivity rises.

Comparing the constant returns-to-scale sectors in the standard GTAP model to the monopolistic competition sector in Melitz reveals three main differences. First, the standard GTAP model is an aggregate industry level framework capturing the behavior of a representative firm in a perfectly competitive industry. In opposite to that, firms face fixed cost of entry in the monopolistic competition framework which leads to increasing return to scale. Second, the standard GTAP model with one representative firm in each industry cannot reflect productivity differences as depicted in the Melitz model. Third, in the standard GTAP model, consumer's utility is defined by an Armington composite of goods from different origins, while in monopolistic completion models it is defined over the Dixit-Stiglitz (1977) composite of varieties differentiated by origin

which allows for monopolistic competition between firms operating on the same trade link. The combination of these differences allows depicting the extensive margin of trade in CGE models with firm heterogeneity.

### 3.1. Algebraic representation of firm heterogeneity in CGEBox

This section presents an algebraic representation of the implementation of the Melitz (2003) model as implemented in CGEBox. The Melitz framework focuses on intra-industry differentiation where each firm produces a single unique variety. However, data at the firm level are limited and applied equilibrium models work at rather aggregate industry levels. Fortunately, Melitz offers a numerical framework build around (marginal changes of) the average firm operating on a trade link. That average firm's productivity comprises all necessary information on the distribution of productivity levels of firms active on that link. That vastly eases the model's implementation by effectively eliminating any data needs at individual firm level as detailed below. Against the background of that definition of an average firm on each trade link, we now focus on the formulation of an empirically computable version of Melitz model and its linkages with the GTAP model.

Assume that a representative agent  $a$  (private households, government, investors, intermediate inputs by the different firms) in region  $s$  obtains utility  $U_{aais}$  from consumption of the range of differentiated varieties of product  $i$ . Considering the constant elasticity of substitution (CES) utility function as proposed by Dixit and Stiglitz (1997), the aggregate demand by each agent  $a$  for commodity  $i$  in region  $s$  ( $Q_{ais}$ ) which is equivalent to utility ( $Q_{ais} \equiv U_{ias}$ ) can be represented as

$$Q_{ais} = \left( \sum_r \int_{\omega \in \Omega_{irs}} \lambda_{airs} \frac{1}{\sigma_{is}} Q_{airs}(\omega) \frac{\sigma_{is}-1}{\sigma_{is}} d\omega \right)^{\frac{\sigma_{is}}{\sigma_{is}-1}} \quad (1)$$

where  $\Omega_{irs}$  represents the set of products  $i$  sourced from region  $r$  to  $s$  and  $\omega \in \Omega_{irs}$  index the varieties in the set  $\Omega_{irs}$ . In this context,  $Q_{airs}(\omega)$  represents the demand quantity of commodity  $i$  for variety  $\omega$  in region  $s$  by agent  $a$  which is sourced from region  $r$ ,  $\sigma_i$  represents the constant elasticity of substitution for each commodity, and  $\lambda_{airs}$  are preference weights (share parameters)<sup>4</sup> which reflect differences between origins not linked to diversity in varieties. Note that substitution elasticities might be differentiated by destination region  $s$ , but are uniform across agents in each region in our implementation.

The resulting CES unit expenditure function which is the dual price index on Dixit-Stiglitz composite demand in region  $s$  ( $P_{ais}$ ) is given by

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<sup>4</sup> The reader should note that the share parameters are absent in the original Melitz paper. We hence allow here a differentiation between products from different origins as in the Armington model in addition to the love of variety effect.

$$P_{ais} = \left( \sum_s \int_{\omega \in \Omega_{irs}} \lambda_{airs} PA_{airs}(\omega)^{1-\sigma_{is}} d\omega \right)^{\frac{1}{1-\sigma_{is}}} \quad (2)$$

where  $PA_{airs}(\omega)$  is agent's a (purchase) price of product  $i$  for variety  $\omega$  in region  $s$  sourced from region  $r$ . Using the aggregate price index in Melitz (2003) based on his definition of the average firm and considering that varieties do not differ in their marginal utility for the first unit, one can define the price index as equivalent to the dual price defined in (2)

$$P_{ais} = \left( \sum_r \lambda_{airs} N_{irs} \widetilde{PA}_{airs}^{1-\sigma_{is}} \right)^{\frac{1}{1-\sigma_{is}}} \quad (3)$$

where  $\widetilde{PA}_{irs}$  denotes the agent price inclusive of export, import and consumption taxes for the average firm, and  $N_{irs}$  refers to the number of firms operating on the trade link  $r$ - $s$ . Note that consistent with Melitz (2003), there is a one to one mapping between firms and varieties such that the number of firms is equal to the number of varieties on each trade link. Comparing to (2), which is based on the individual varieties, (3) summarizes the compositional change, i.e. change in the number of varieties which go along with an update of the average price. Note again that we assume the same substitution elasticities across agents

The total ( $Q_{airs}$ ) and average per firm ( $\widetilde{Q}_{airs}$ ) demand for the average variety by an agent to be shipped from  $r$  to  $s$  ( $\widetilde{Q}_{airs}$ ) can be obtained by applying *Shephard's Lema* on the expenditure function:

$$Q_{airs} = \widetilde{Q}_{airs} N_{irs} = \lambda_{airs} N_{irs} Q_{ais} \left( \frac{P_{ais}}{\widetilde{PA}_{airs}} \right)^{\sigma_{is}} \quad (4)$$

Which reveals the main differences to a standard Armington composite: the share parameters vary with the number of operating firms, i.e. the number of varieties comprised in the bilateral trade bundles. As the agent demand for the average firm's output in region  $r$  in each industry  $i$  in region  $s$  ( $Q_{airs}$ ) depends on the aggregate regional demand for that industry  $Q_{ais}$ , we need to determine this in equilibrium for each agent. In other word, we need to determine the demand for use of  $i$  as an intermediate input and as final demand for household consumption, government consumption, investment, and for international transport margins. In the standard GTAP model, each agent has a specific preference function which determines the demand for her Armington commodity; the government and saving sector based CES preferences while households used a CDE indirect demand function. The Armington demand for each agent and commodity is then decomposed into a domestic and import component in a first Armington nest. The second one decomposes import demand by each region by origin, independent of the agent.

The implementation of the Melitz model thus simplifies the demand structure present in the standard GTAP model by aggregating the two Armington nests into a single one, however, note that the GTAP data base so far does not differentiate in the SAM bi-lateral flows by agent. We hence used the same shares by origin to split up import demand for the different agents. Alternatively, MRIO shares from the OECD Metro model can be used to allow for differentiated shares.

Assume a small profit maximizing firm facing the constant elasticity of demand according to (4) for its variety. Based on the assumption of the large group monopolistic competition, firms will not consider its impact on the average price index and therefore follow the usual markup rule to translate their marginal cost of production to the optimal price.

Firms in Melitz (2003) face different types of cost: sunk fixed cost of entry  $f^{ie}$ , fixed cost of operating on a trade link  $f_{irs}$  and marginal cost  $c_{ir}$ . Let  $\varphi_{irs}$  indicate the firm's specific productivity which measures the amount of "variable composite unit" needed per unit of output  $Q_{irs}$ . Accordingly, the marginal cost per unit is the amount of "composite input" required per  $(\frac{1}{\varphi_{irs}})$  times the unit cost of the "variable composite input" ( $c_{ir}$ ) in industry  $i$  of region  $r$ . Therefore, a firm wishing to supply  $Q_{irs}$  units from region  $r$  to  $s$  employs  $(f_{irs} + \frac{Q_{irs}}{\varphi_{irs}})$  units of "variable composite input". Let,  $\tau_{irs}$  denote the fixed iceberg cost of trade which represent domestic production costs, and not the international trade margins present in GTAP. Focusing on the average firm with a productivity  $\tilde{\varphi}_{irs}$  operating on a trade link, and solving the firm's profit maximization problem, the price charged by the average firm in region  $r$  to supply region  $s$   $\widetilde{PF}_{irs}$  (inclusive of domestic transport margin) is

$$\widetilde{PF}_{irs} = \frac{\sigma_{is}}{\sigma_{is} - 1} \frac{\tau_{irs} c_{ir}}{\tilde{\varphi}_{irs}} \quad (5)$$

where  $\frac{\sigma_{is}}{\sigma_{is}-1}$  represents the constant markup ratio in industry  $i$  which reflects market power due to product differentiation into varieties. The linkages between the firm and agent prices are provided in Appendix 1.

The average price in (5) depends hence on the price of variable composite input  $c_{ir}$ , which is a function of the price of intermediates and primary factors. Given the assumption of constant return to scale and the way technology is presented in the standard GTAP model, the unit cost function for sector  $i$  in region  $r$   $c_{is}$  in GTAP is given by the Leontief composite of the value added bundle (CES aggregate of factors of production) and the aggregate of intermediate demand (Leontief aggregate of intermediate demands). In the CGEBox,  $m\_px$  is a macro defined as producer price which constitute per unit costs corrected for production taxes. To be consistent with our Melitz formulation, the unit cost inclusive of production tax is directly introduced in the markup equation (5). It should be emphasized that the presence of fixed cost in the Melitz model

is the source of increasing returns to scale in a monopolistically competitive industry: if firms expand production, the fixed cost can be distributed over a larger outputs such that per unit cost decrease.

While observed data on quantities traded and related prices allow identifying the necessary attributes on the average firm, additional information is needed to gain information about the marginal firm, i.e. the firm which earns zero profit. Obviously, the distance in productivity between the average and marginal firm reflects properties of the underlying distribution. We rely here on a Pareto Productivity distribution which has analytical tractability (Chaney, 2008) and was shown as empirically relevant (Axtell, 2001; Luttmer, 2002; Gabaix, 2008; Eaton et al., 2001).

Let  $M_r$  indicate the number of firms choosing to incur the fixed entry cost, i.e. total industry size, each individual firm receives its productivity  $\varphi$  draws from a Pareto distribution with Probability Density Function (PDF)

$$g(\varphi) = a \left( \frac{b^a}{\varphi^{a+1}} \right) = \frac{a}{\varphi} \left( \frac{b}{\varphi} \right)^a \quad (6)$$

and Cumulative Distribution Function (CDF)

$$G(\varphi) = 1 - \left( \frac{b}{\varphi} \right)^a \quad (7)$$

where  $b$  is the minimum productivity and  $a$  is a shape parameter. Lower values of the shape parameter imply higher productivity dispersion among firms. As discussed in Melitz (2003),  $a > \sigma_i - 1$  should be applied in order to ensure a finite average productivity level in the industry.

On each bilateral trade link, the given the fixed bilateral trade cost, variable costs and demand define jointly a certain cut off productivity level ( $\varphi_{rs}^*$ ) at which firms will receive zero profit. A firm which has drawn the productivity equal to that threshold level ( $\varphi_{rs} = \varphi_{rs}^*$ ) will hence face zero profits and act as the marginal firm from region  $r$  supplying  $s$ . Those firm whose productivity is above the threshold level ( $\varphi_{rs} > \varphi_{rs}^*$ ) will receive a positive profit and will operate on the  $r - s$  link and those firm whose productivity is below the threshold level ( $\varphi_{rs} < \varphi_{rs}^*$ ) will not operate on the  $r - s$  link. Focusing on the fixed operating cost  $f_{irs}$  in composite input units, the marginal firm on r-s link receives zero profit at

$$c_{ir} f_{irs} = \frac{r(\varphi_{irs}^*)}{\sigma_i} \quad (8)$$

where  $r(\varphi_{irs}^*) = p(\varphi_{irs}^*)q(\varphi_{irs}^*)$  denotes the revenue of marginal firm at the productivity equal to the cut off level ( $\varphi_{irs} = \varphi_{irs}^*$ ).

The zero cut off productivity level in each bilateral market  $\varphi_{irs}^*$  can be obtained by solving (8). ~~888888~~ However, it is numerically easier to define this condition in terms of the average rather than the marginal firm. To do this, we define the productivity and revenue of the average firm relative to that of the marginal firm.

The average productivity in a trade link is determined by the productivity level of the operating firms on that link which by definition are at or above the cutoff productivity level. Following Melitz (2003), that average productivity is defined as the CES aggregation of productivities of all firms operating on a given trade link

$$\tilde{\varphi}_{irs} = \left[ \frac{1}{1 - G(\varphi_{irs}^*)} \int_{\varphi_{irs}^*}^{\infty} \varphi_{irs}^{\sigma_{is}-1} g(\varphi) d\varphi \right]^{\frac{1}{1-\sigma_{is}}} \quad (9)$$

If these productivities are Pareto distributed, we can write<sup>5</sup>

$$\tilde{\varphi}_{irs} = \left[ \frac{a}{(a + 1 - \sigma_{is})} \right]^{\frac{1}{1-\sigma_{is}}} * \varphi_{irs}^* \quad (10)$$

Eq. (10) provides the relationship between the productivities of the average and marginal firm (for further details see Allen and Arkolakis (2016)).

Using optimal firm pricing according to (5) and given the input technology, the ratio of revenues of the firms with marginal productivity  $r_{irs}(\varphi^*)$  in relation to the revenue of the firm with the average productivity  $r_{irs}(\tilde{\varphi})$  is defined as

$$\frac{r_{irs}(\varphi^*)}{r_{irs}(\tilde{\varphi})} = \left( \frac{\varphi^*}{\tilde{\varphi}} \right)^{\sigma_i} \quad (11)$$

Solving (10) for  $\frac{\varphi^*}{\tilde{\varphi}}$ , substituting it into (11), and then solving the resulting equation for  $r_{irs}(\varphi^*)$  and replacing its value in (8), defines a relation between the bilateral fixed cost at current composite input price (the left hand side of (12) below), the average firms revenue ( $\tilde{P}\tilde{F}_{irs} \tilde{Q}_{irs}$ ), the shape parameter of the Pareto distribution of the productivities and the substitution elasticity of demand:

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<sup>5</sup> One could use industry specific shape parameter ( $a_i$ ) given the availability of data at sectoral level. In this study we assume that all firms entering in different industries draw their productivity from the Pareto distribution function with same characteristics (i.e. same scale and shape parameter).

$$c_{ir}f_{irs} = \frac{(a + 1 - \sigma_{is})}{a\sigma_{is}} \widetilde{PF}_{irs} \widetilde{Q}_{irs} \quad (12)$$

Note that average firm's sale in region  $r$  in each industry  $i$  to region  $s$  ( $\widetilde{Q}_{irs}$ ) at the equilibrium is composed of the demand for use of  $i$  by different agents<sup>6</sup>.

The optimal pricing in the markup equation (5) requires information on the average productivity on each bilateral trade link. In Melitz (2003), the probability that a firm will operate is  $1 - G(\varphi_{irs}^*)$  which is equal to the fraction of operating firms over total number of firms choosing to draw their productivity ( $\frac{N_{ir}}{M_{ir}}$ ). Using the Pareto cumulative distribution function (7) and inverting it we have

$$\varphi_{irs}^* = \frac{b}{\left(\frac{N_{irs}}{M_{ir}}\right)^{\frac{1}{a}}} \quad (13)$$

substituting (13) into (10) results in

$$\widetilde{\varphi}_{irs} = b \left[ \frac{a}{(a + 1 - \sigma_{is})} \right]^{\frac{1}{1 - \sigma_{is}}} * \left(\frac{M_{ir}}{N_{irs}}\right)^{-\frac{1}{a}} \quad (14)$$

Next, the number of firms selecting to enter the market  $M_{ir}$  is determined. Based on the free entry condition, the last firm which enters has expected profits over its life time which just offset the sunk cost of entry. Industry entry of a firm requires a one-time payment of  $f^{ie}$ . An entered firm faces a probability of  $\delta$  in each future period to suffer a shock which forces its exit. Therefore,  $\delta M_{ir}$  firms are lost in each period. Based on Melitz (2003), in a stationary equilibrium, the number of aggregate variables must remain constant over time, including industry size. This requires that the number of new entrants in every period is equal to the number of firms lost  $\delta M_{ir}$ . Therefore, total entry cost is equal to  $c_{ir} \delta M_{ir} f^{ie}$ . Each firm faces the same expected share on that cost, i.e.  $c_{ir} \delta f^{ie}$  if risk neutral behavior and no time discounting is assumed. The firm's expected share on entry costs must be equal to the flow of expected profit on the condition that firm will operate.

$$\widetilde{\pi}_{irs} = \frac{\widetilde{PF}_{irs} \widetilde{Q}_{irs}}{\sigma_{is}} - c_{ir}f_{irs} \quad (15)$$

The probability that a firm will operate on the  $r$ - $s$  trade link is given by the ratio of  $\frac{M_{ir}}{N_{ir}}$ <sup>7</sup>. Thus, the free entry condition ensures that the expected industry profits, i.e. the profits summed up over all potential bilateral trade links, is equal to the annualized flow of the fixed costs of entry

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<sup>6</sup>  $\widetilde{Q}_{irs} = \sum_a \widetilde{Q}_{airs}$

$$c_{ir} \delta f^{ie} M_r = \sum_s N_{irs} \bar{P}_{irs} \tilde{Q}_{irs} \frac{\sigma_{is} - 1}{a\sigma_{is}} \quad (16)$$

where zero profit condition (12) is used to replace the fixed operating cost  $c_{ir}f_{irs}$ .

With the number of entered firm established in (16), we now turn to total composite input demand of the industry Y which consists of three components: sunk entry costs of all entrants ( $\delta M_{ir} f^{ie}$ ), operating fixed cost ( $\sum_s N_{irs} f_{irs}$ ) on each trade link and variable costs ( $\sum_s N_{irs} \frac{\tau_{irs} \tilde{Q}_{irs}}{\tilde{\varphi}_{irs}}$ ). Therefore, composite input demand is defined as

$$Y_{ir} = \delta M_{ir} f^{ie} + \sum_s N_{irs} (f_{irs} + \frac{\tau_{irs} \tilde{Q}_{irs}}{\tilde{\varphi}_{irs}}) \quad (17)$$

This equation provides the link to the equations in the GTAP model describing the technology and related costs. Table 1 summarizes full set of Melitz equations which are introduced into the GTAP model.

**Table 1: Equilibrium conditions**

Equation	Equilibrium condition	Associated variable
(3)	Sectoral Aggregation	$P_{is}$ : Sectoral price index
(4)	Firm-level demand	$\bar{P}_{irs}$ : Average firm price
(5)	Firm-level Pricing	$\tilde{Q}_{irs}$ : Average firm quantity
(12)	Zero cut off condition	$M_{ir}$ : Number of operating firms
(14)	CES Weighted Average productivity	$N_{ir}$ : Average firm productivity
(16)	Free entry condition	$N_{irs}$ : Number of entered firm
(17)	Factor market clearing condition	$c_{ir}$ : Sectoral composite input price

<sup>7</sup> The probability that the firm will operate is  $1 - G(\varphi_{irs}^*) = \frac{N_{ir}}{M_{ir}}$

Variables through which Melitz model is linked to the GTAP
$Q_{ais}$ : Sectoral Demand
$c_{ir}$ : Price of composite input
$Y_{ir}$ : Sectoral composite input demand

### 3.2. Calibration of the model

In order to apply the above equation structure, the different parameters must be chosen such as to recover an observed benchmark. That benchmark consists of the global SAM provided by the GTAP data base<sup>8</sup> against which the remaining equations of the GTAP model are calibrated as well. It comprises values on domestic sales and on bilateral international trade expressed in USD million for each sector and region which are key data in the context of the Melitz module discussed above. Global detailed SAMs comprise many small entries both in relative and absolute terms which can affect the numerical stability during solution of a CGE. We therefore recommend filtering out in a systematic way small transactions in relative terms from the database when using the model with many sectors and regions, following by rebalancing the global SAM (see Britz and Van der Mensbrugge 2016).

We focus here mainly on the calibration of the firm heterogeneity module while the calibration of Armington sectors are similar to the standard GTAP model and are not further discussed. In the following, a superscript “0” denotes a benchmark value, for instance,  $Q_{is}^0$  denotes the benchmark value of demand for commodity  $i$  in region  $s$ . In order to line up the variables in the Melitz module with the SAM, the following identities must hold:

$$c_{ir}^0 Y_{ir}^0 \equiv vom_{ir} \quad (18)$$

$$P_{is}^0 Q_{ais}^0 \equiv xafm_{ais} \quad (19)$$

where  $vom_{ir}$ , represent the production value of commodity  $i$  in region  $r$  at producer tax inclusive prices and  $xafm_{airs}$  represent the value of each agent’s demand for commodity  $i$  in region  $s$ , consumer tax inclusive. The first identity indicates that the cost of input supply must equal the value of output, and the second identity ensures that demand for all goods and factors is equal to supply at the benchmark. Accordingly, using the conventional choice of unity prices, i.e.  $P_{irs}^0 = 1$ , and  $c_{irs}^0 \equiv 1$ , total quantity demanded  $Q_{is}^0$  and total input supplied  $Y_{ir}^0$  is locked down.

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<sup>8</sup> We used the GTAP8 database (Narayanan and McDougall 2012) for the results presented later which carries a snapshot of the 2007 world economy, covering 129 regions (aggregate of 226 countries) and 57 sectors. But all GTAP data bases share the basic structure such that the code can also be used with other releases such as the GTAP9 data base recently used by us which refers to 2011.

It should be noted here again that the bilateral import demand in the SAM is aggregated over agents. The RHS entries in equation (19) above are hence constructed by splitting up that total by each agent's share on total imports.

Given the agents' demand for each commodity  $Q_{ais}^0$  and using the definition of the firm average in Melitz, we have

$$Q_{ais}^0 = \sum_r Q_{airs}^0 = \sum_r \tilde{Q}_{airs}^0 N_{irs}^0 \quad (20)$$

We now briefly compare the demand side based on the Armington assumption with the Melitz model to demonstrate that the choice of  $N_{irs}^0$  does not matter for the model's simulation behavior. Under the Armington assumption, a CES utility function is used to differentiate between origins. That implies that the (average) quality on the trade links can differ along with the price. As the resulting demand function is homothetic, expenditure shares are independent of the income level and solely depend on relative prices for these qualities. Conveniently, calibration is performed at given substitution elasticities by choosing share parameters such that given expenditure shares are recovered at given relative prices.

The Ditz-Stieglitz price index used in the Melitz (2003) is also based on a CES utility function; however, here we have a continuum of varieties. As share parameters are absent in the original model, the marginal utility of the first unit for each variety is the same. Each firm is assumed to produce its own variety and multiple firms are allowed to operate on a trade link. Thus, more firms imply more varieties and a higher utility per unit of traded output on that link. Total demand on a link is hence defined as the product of average output per operating firm and the number of firms operating on that link. Recovering expenditures shares at given prices for each trade link can hence be based either by deriving the number of firms on a link at given average firm output on that link or by pre-selecting the number of firms for each link and deriving average outputs. Share parameters are not needed; however, as a consequence, the resulting price index cannot be controlled. As only relative prices matter, that only affects readability and not simulation behavior.

In order to improve readability and provide a combined interpretation of the two models, we introduce share parameters in the CES-demand function used in the Melitz model. That allows a convenient interpretation of the extension introduced by the Melitz model: changes in the number of firms lead to preference shifts in the Armington model as they update the share parameters. The resulting price index  $P_{ais}$  and matching demands  $\tilde{Q}_{airs}$  for product  $i$  in region  $s$  from origins  $r$  by agent  $a$  can be depicted as

$$P_{ais}^0 = \left( \sum_r \lambda_{airs} \frac{N_{irs}}{N_{irs}^0} \tilde{P}A_{airs}^0 \right)^{\frac{1}{1-\sigma_{is}}} \quad (21)$$

$$Q_{airs}^0 = \lambda_{airs} \frac{N_{irs}}{N_{irs}^0} Q_{ais}^0 \left( \frac{P_{ais}^0}{\widetilde{P}A_{airs}^0} \right)^{\sigma_{is}} \quad (22)$$

where  $N_{irs}$  depicts the number of operating firms on a trade link. Note that, as  $N_{irs}$  and  $\lambda_{airs}$  (the share parameter) are identical in the calibration point, we scale without any impact on the simulation behavior the lambdas  $\lambda_{airs}$  such that the price index  $P_{is} \equiv 1$  (or something else) at the benchmark. That allows to recover any given quantity index  $Q_{ais}$  and matching  $P_{ais}$ . Equations 21 and 22 show the link between the Armington and Melitz models. A given Armington model can hence be simply extended by adding changes in the number of operating firms. With the choice of  $\widetilde{P}A_{airs}^0 = 1$ ,  $P_{ais}^0 = 1$ , since we now have the observed value of  $Q_{airs}^0$ ,  $Q_{ais}^0$ , and estimated value  $\sigma_{is}$ <sup>9</sup>, the value of  $\lambda_{airs}$  is calibrated in a way that we do not need the information on the benchmark number of firms. Accordingly, the values of  $\lambda_{airs}$  are recovered by inverting the demand functions.

$$\lambda_{airs} = \frac{Q_{airs}^0}{Q_{ais}^0} \left( \frac{\widetilde{P}A_{airs}^0}{P_{ais}^0} \right)^{\sigma_{is}} \quad (23)$$

Let  $\widetilde{P}F_{irs}$  denote the average firm's offer price on a specific trade link, from which  $\widetilde{P}F_{airs}$  is derived by considering export taxes (to arrive at cif), international transport margins (to arrive at fob), and import and consumption taxes.

The composite input demand linked to bilateral fixed cost  $f_{irs}$  is derived from the zero-profit cut off condition, where the values of  $c_{ir}^0$  and  $\widetilde{P}F_{irs}^0$  are set to unity at the benchmark,  $\widetilde{Q}_{irs}$  is estimated from **(Fehler! Verweisquelle konnte nicht gefunden werden.)**, and the value of shape parameter  $a$  is taken from literature<sup>10</sup>:

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<sup>9</sup> In GTAP the first level Armington demand for each agent  $XA_{i,h}$ ,  $XA_{i,gov}$ ,  $XA_{i,inv}$  is a CES composite of domestic and aggregate imported good with the substitution elasticity  $\sigma_m$ ; and the second level Armington demand (aggregated import) is a CE's composite of import demand by each region of origin with the substitution elasticity of  $\sigma_w$ . However, in Melitz structure represented consume has Dixit –Stiglitz preferences over the varieties including domestically produced commodities and imported commodities by source of origin with the substitution elasticity of  $\sigma_i$ . To be consistent with Dixit –Stiglitz framework,  $\sigma_i$  is derived based on the weighted average of the substitution elasticities of first and second level Armington nests where the weights given to  $\sigma_m$  and  $\sigma_w$  are domestic and import purchases at agent price, respectively. We also insured that derived substitution elasticity is smaller than  $a + 1$  to ensure a finite average productivity level in the industry.

<sup>10</sup> Estimates of the value shape parameter vary and are conditional on a choice of elasticity of substitution. The choices of shape parameter and substitution elasticity are important as these key parameters have significant implications (for example on welfare). The importance of these variables is well discussed in Akgul et al. (2015, 2016) and Dixit (2016). Bernard et al. (2007) choose a shape parameter equal to 3.4, and estimates of Eaton et al. (2004) show  $a = 4.2$ , while Balistreri et al. (2011) find shape parameter of 5.17, 4.58 and 3.92 depending on different trade cost-distance elasticities but under the maintained assumption that  $\sigma = 3.8$ . Akgul (2016) calibrated shape parameter of 2.89 for manufacturing sector consistent with a shape parameter obtained Spearot (2016). In this study we used the weighted average of the substitution elasticities of first and second level Armington nests which is not necessary consistent with the shape parameter (4.6) which is taken from Balistreri et al.(2011). Once decided

$$f_{irs} = \frac{\widetilde{PF}_{irs}^0 \widetilde{Q}_{irs}^0}{c_{ir}^0} \frac{(a + 1 - \sigma_i)}{a\sigma_i} \quad (24)$$

Please note that  $Q_{irs}^0 = \widetilde{Q}_{irs}^0 N_{irs}^0$  implies that changing the number of firms at the benchmark updates  $f_{irs}$ , but not the total industry cost linked to bi-lateral fixed cost.

Similarly, the free entry condition allows deriving the composite input demand of sunk entry cost  $c_{ir} \delta f^{ie}$  at given price and quantity of the average firm:

$$c_{ir} \delta f^{ie} = \sum_s \frac{N_{irs}^0}{M_r^0} \widetilde{PF}_{irs}^0 \widetilde{Q}_{irs}^0 \frac{\sigma_{is} - 1}{a\sigma_{is}} = \sum_s \frac{1}{M_r^0} \widetilde{PF}_{irs}^0 Q_{irs}^0 \frac{\sigma_{is} - 1}{a\sigma_{is}} \quad (25)$$

Note again that the given values on the trade links  $Q_{irs}^0$  determine together with the chosen industry size the sunk entry costs such that, as above, either the number of operating firms or the average quantities can be chosen without affecting the calibration. Furthermore, the condition reveals that the choice of total industry size  $M_r^0$  does not matter either, as the sunk entry costs will adjust proportionally such that the industry's total annualized costs of industry entry solely depend on the shape parameter of the Pareto distribution, the substitution elasticity and the given SAM values.

Next, given a minimum productivity parameter of the Pareto distribution  $b$ , the average firm productivity on trade link can be initialized as:

$$\widetilde{\varphi}_{irs} = b \left[ \frac{a}{(a + 1 - \sigma_{is})} \right]^{\frac{1}{1 - \sigma_{is}}} * \left( \frac{M_r^0}{N_{irs}^0} \right)^{-\frac{1}{a}} \quad (26)$$

Having determined the value of average productivity on each r-s link, and setting  $c_i \equiv 1$ ,  $\widetilde{PF}_{irs}^0 \equiv 1$ , the domestic input demand related to the transport margin cost can be computed by inverting the mark up equation (5) as follows:

$$\tau_{irs} = \frac{\sigma_{is} - 1}{\sigma_{is}} * \widetilde{\varphi}_{irs}^0 \quad (27)$$

Here, it is less obvious why  $M_r^0$  and  $N_{irs}^0$  can be freely chosen without affecting simulation behavior.  $\widetilde{\varphi}_{irs}$  and  $\tau_{irs}$  enter the total industry cost and the price markup equations. Total industry costs on a trade link r-s are given as:  $N_{irs} \left( f_{irs} + \frac{\tau_{hrs} \widetilde{Q}_{irs}}{\widetilde{\varphi}_{hrs}} \right)$  which shows that changes in

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which sectors are treated as sectors with heterogeneous firms, the theoretically consistent sectoral elasticity of substitution and sectoral shape parameter should be obtained. The minimum productivity parameter is chosen as  $b = 0.2$  following Balistreri et al. (2011) from Bernard et al. (2007).

$\tilde{\varphi}_{irs}^0$  and  $\tau_{irs}$  resulting from different benchmark values of  $N_{ir}$  and  $M_r$  do not matter. The same holds for the price markup equation:  $\widetilde{PF}_{irs} = \frac{\sigma_{is}}{\sigma_{is}-1} \frac{\tau_{irs} c_{ir}}{\tilde{\varphi}_{irs}}$ .

Accordingly, sensitivity analysis with different values for  $M_r^0$  and  $N_{irs}^0$  showed no differences in simulated results. However, the choice might affect numerical stability by affecting the overall scaling of the model. In order to improve readability, we have generally chosen  $N_{irs}^0 \equiv 1$  which gives average firm outputs on each trade link equal to given bi-lateral respectively domestic sales values at benchmark prices.

### 3.3 Technology nesting

Similar to Zeynep et al. 2016 in their GTAP-HET model, we apply a different nesting for variable costs of trade and the fixed costs related to industry entry and operating on a link. The variable costs maintain whatever nesting is chosen originally by the analyst. The fixed cost only use fixed value added, applying the same substitution elasticity as used for the value added nest in the original model. Note that this is a first implementation which should be improved as it implies for industries using land or natural resources that they also become part of the fixed costs. Technically, the flexible nesting structure by CGEBox is applied by defining a new technology nest termed “fcost”. At the same time, the implementation of the nesting was extended to allow for the case that several nests demand the same intermediate input or factor. Alternatively, the model can be simplified to use the same composite input for variable and fixed costs. The implementation allows combining the default Melitz model with its differentiation between variable and fixed costs composites with other nesting structures on the production side such as the implementation of GTAE-E or GTAP-AGR realized as modules of CGEBox.

### 3.4 Numerical stability and domestic industry demand for its own output

Test runs with the model in different configurations with regard to sectoral and regional detail as well as differently deep multi-lateral trade liberalization shocks have consistently shown non-stable behavior resulting in infeasibilities in cases where an industry’s cost share of domestic intermediate input is relatively high, say around 30%. Under these conditions, the model easily ends up in a vicious circle where a cost increase on the domestic link amplifies itself such that corner solutions with zero industry sizes provoke infeasibilities. Changing the parameterization seems to help only in some selected cases.

A firm should not apply mark-up pricing for own produced and used intermediates if that decreases its own competitiveness, i.e. provokes real costs and is not a tax evasion tactic. However, the average firm model does not differentiate domestic intermediate demand inside a sector between a firm’s demand for its own produce and demand by other firms. In order to allow for a numerical stable implementation, we therefore allow excluding domestic intermediate demand for the diagonal I/O element from the love of variety effect, i.e. applying Armington preferences. Accordingly, there is also no markup pricing involved if that option is chosen. That

has proven to effectively prevent the down-spiraling solution behavior in most cases. However, it can still happen if a bundle of sectors shows quite high costs share of domestic demand for the same bundle of sectors. In that case, the defense by arguing over the single firm’s behavior is not longer valid.

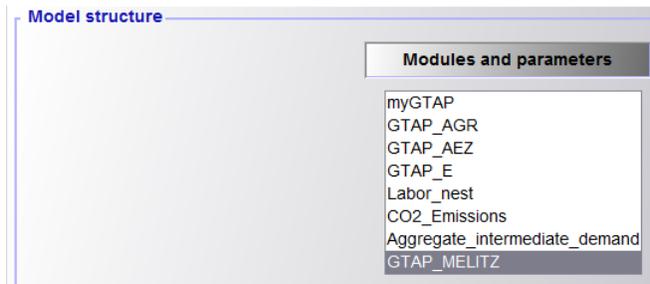
Potential improvements in that regard could encompass a split up of the diagonal domestic intermediate demand into a share subject to the love of variety assumption and a remainder treated as competitive. But before introducing further complexity into the framework, more testing is required.

## 4. Technical implementation and an example application

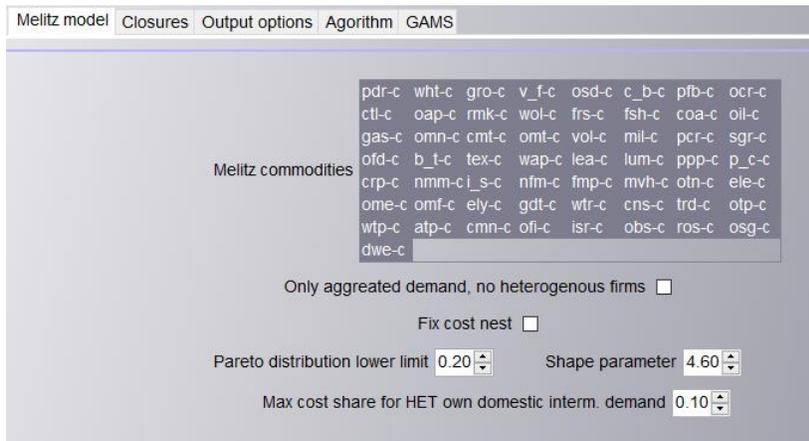
### 4.1 Technical implementation

The necessary GAMS code for the calibration of the model can be found in “gams\GTAPMelitz\GTAPMelitz\_cal.gms”. Besides the calibration steps discussed above, it is important to mention that certain flags are deleted to ensure that equations in the standard model are inactive for Melitz commodities.

The user can switch the module one on the Graphical User Interface under “Model Structure”:



which generates a new tab with the following input possibilities.



Under Melitz commodities, the commodities / sectors can be chosen the parameter of the Pareto-distribution of the productivities defined.

The checkbox “Only aggregated demand, not heterogenous firms” changes the demand structure of the standard GTAP such that only one Armington nest shared by all agents is present. The substitution elasticity is defined as in the case when the full Melitz model is used. The remaining equations of the Melitz model are absent and replaced by the equations of the standard GTAP model. That allows dis-entangling impacts of the full Melitz model from the structural changes on the demand side.

The checkbox “Fix cost nest” introduces a differentiation between variable and fix costs input composites where the latter only comprises primary factors, at least as long as there is some minimal primary factor cost share left in the variable input composite. The spinner termed “Max cost share of HET domestic interm. demand” allows switching off love of variety for domestic intermediate demand by same industry depending on the costs share. Only sector-regional combinations with a share above the threshold will be excluded. Setting the threshold to unity will hence leave the love of variety effect switch on for all domestic intermediate demand, while zero will switch it off on all cases.

Detail on simulated value of variables found in the Melitz module can be found in two tables under the “Trade group” as shown below:



## 4.2 An example application

We use a 50% reduction of all imports tariff and export subsidies globally with a 10x10 aggregation of the GTAP8 data base as an example application<sup>11</sup>, focusing on welfare changes and highlighting the newly available information from the model. We compare the standard GTAP model against two variants, one where only the demand structure of the Melitz model is used and a second one with its full implementation.

Table 2 below reports the Money metric for the full Melitz model (Tariffs\_m), the model where only the demand side of the Melitz model is used (Tariffs\_d) and the Standard GTAP model (Tariffs). First, it can be seen that aggregate welfare is almost not affected by choosing the more simple demand side with only one Armington nest shared by all agents. However, results for individual regions show some sensitivity.

<sup>11</sup> Appendix 2 provides the sectoral and regional aggregation of GTAP sectors into the new mapping.

As expected, adding the full Melitz model increases welfare, here, it more than quadruples the global welfare impact of multi-lateral trade liberalization. Interestingly, that impact is not uniform across regions: whereas North America suffers a small welfare loss in the standard configuration, it gains under the Melitz model. However, the welfare in Middle East and North Africa is the same under two structures. Generally the results are in line with finding of Balistreri et al. (2011) which also found welfare increases around factor four.

Note that, for our simulation exercise, we compare the welfare impact of the policy shock under different structures where more or less the same value of Armington elasticity is assigned in each structure. Specifically, our calibration code restricts the substitution elasticity to an interval of  $\pm 1/2$  around the share parameter which for many sectors will yield more or less the same consumption quantity weighted elasticity as found in the two-level Armington system of GTAP.

Dixon et al. (2016) argues that in order to compare the welfare impacts of a policy shock under Armington structure and Melitz structure one should assign the substitution elasticity ( $\sigma$ ) in the Armington framework such that it simulated trade flows comparable to a Melitz framework. In the two commodity and two country model of Dixon et al. (2016); a substitution elasticity of 8.45 yielded trade flows similar to a Melitz model with a substitution elasticity of 3.8 when tariff in one of the countries increases by 7.18 percent. Dixon et al. (2016) obtained this equivalent Armington value of substitution using a trial and error approach. The substitution elasticity in an Armington model which yields similar results depends not only on the parameterization of the Melitz model, but also on the type and magnitude of the shock and structure of the model as a whole. Using trial and error to an “equivalent” substitution elasticity to replicate the trade pattern after a given shock can only be used if the model is rather small. However, with a medium sized model, we found that once  $\sigma$  is adjusted in Armington framework to replicate one of the trade flows, the error in other bilateral trade flows might even become larger. Such finding is in line with that of Balistreri et al. (2011). Akgul (2015, 2016), who introduced a theoretically consistent framework from which one can calibrate the sector specific Armington elasticity given the obtained sector specific value of the shape parameter and econometrically estimated results. Given data limitations, we adjust the given Armington elasticities around a uniform Pareto shape parameter across regions and sectors. This assumption remains open to critic and the choices of theoretically consistent substitution elasticities across regions and sectors is required in empirical analysis.

**Table 2: Money metric in comparison of Melitz and Standard GTAP model**

Money metric decomposition [0]				
Sectors and institutions		Total	Years shock	
Total [Bio \$]	Total			
Tariffs	Tariffs_d	Tariffs_m		
World	44.88	50.89	177.64	
Australia, New Zealand	1.63	1.77	4.23	
East Asia	30.14	31.72	94.15	
Southeast Asia	2.93	4.13	13.64	
South Asia	3.08	1.34	10.19	
North America	-3.76	-0.62	10.96	
Latin America	0.80	0.23	5.01	
European Union 25	4.50	9.21	23.92	
Middle East and North Africa	0.53	-0.02	0.53	
Sub-Saharan Africa	0.68	0.35	1.41	
Rest of World	4.34	2.77	13.60	

The next table 3 shows the simulated overall price index and seems to indicate, as expected, that the Melitz model tends to amplify the impacts found under Armington model.

**Table 3: Aggregate price index of the Armington agents**

Income, compare across regions [0]				
Total or household groups		Sectors and institutions	Years shock	Percentage diff. to Scenarios no Shock
Total	Price index [Index]			
Tariffs	Tariffs_d	Tariffs_m		
World	0.99 -0.62%	0.99 -0.89%	0.99 -1.01%	
Australia, New Zealand	1.00 0.10%	1.00 0.12%	1.00 0.02%	
East Asia	1.01 0.68%	1.01 0.54%	1.01 0.61%	
Southeast Asia	1.00 -0.01%	1.00 0.03%	1.00 0.18%	
South Asia	0.99 -1.36%	0.98 -2.17%	0.98 -2.27%	
North America	1.00 -0.48%	1.00 -0.38%	0.99 -0.67%	
Latin America	0.99 -0.79%	0.99 -0.86%	0.99 -1.12%	
European Union 25	1.00 -0.45%	1.00 -0.28%	0.99 -0.50%	
Middle East and North Africa	0.98 -1.80%	0.98 -2.08%	0.98 -2.35%	
Sub-Saharan Africa	0.98 -1.74%	0.98 -2.07%	0.98 -2.41%	
Rest of World	0.99 -0.93%	0.99 -1.07%	0.99 -1.29%	

The impacts on factor income corrected for changes in the overall price index are less clear with more diversified changes across the regions. In an environment with multiple factors, effects on factor prices differ which seems significant in our application.

**Table 4: Factor income, corrected with price index aggregate Armington agent**

Income, compare across regions [0]				
Total or household groups		Sectors and institutions		Percentage diff. to
Total		Factor income, normalized with price index [Bio \$]		Scenarios
Tariffs		Tariffs_d	Years shock	no Shock
World	39756.41 0.69%	39861.78 0.95%	39922.23 1.11%	
Australia, New Zealand	759.26 0.55%	759.35 0.56%	760.90 0.76%	
East Asia	7226.88 0.69%	7223.42 0.64%	7268.05 1.26%	
Southeast Asia	1081.11 1.23%	1081.91 1.30%	1089.57 2.02%	
South Asia	1242.61 1.52%	1239.76 1.29%	1242.17 1.48%	
North America	12335.02 0.10%	12337.88 0.12%	12345.46 0.18%	
Latin America	2051.05 0.63%	2050.54 0.60%	2054.21 0.78%	
European Union 25	10260.98 0.11%	10266.71 0.17%	10276.61 0.27%	
Middle East and North Africa	448.36 1.67%	447.76 1.53%	447.79 1.54%	
Sub-Saharan Africa	682.86 1.39%	682.22 1.30%	682.14 1.29%	
Rest of World	3561.89 0.76%	3559.80 0.70%	3566.61 0.69%	

Table 5 below summarizes main variables found in the Melitz module. As indicated in the table, as expected, the total number of firms entered (number of domestic verities) decrease as result of liberalization while the number of operating firm indicating the sum of verities consumed increases.

**Table 5: Example summary information from Melitz model**

Melitz, overview [0]				
Exporter		Products		Percentage diff. to
Australia, New Zealand		Heavy Manufacturing		Scenarios
Tariffs_m		Years shock		no Shock
All firms entered	0.98 -2.12%	1.00		
All firms operating	0.05 0.65%	0.05		
Variable cost price	1.00 0.09%	1.00		
Fix cost price	1.00 0.09%	1.00		
General fixed cost	27.51 -2.03%	28.08		
Bi-lateral fixed cost	4.75 -0.57%	4.77		
Variable cost	126.57 -2.03%	129.19		
General fixed cost per unit	0.13 0.37%	0.13		
Bi-lateral fixed cost per unit	0.02 1.86%	0.02		
Variable cost per unit	0.58 0.37%	0.58		

The last table 6 below details information on each trade link. Note first the impact on the domestic sales: as the firm's price in the domestic market drops, the number of firms operating on

the domestic link is reduced and average productivity and output per firm increases. In opposite to that, the firms prices of selling to “South Asia” increase by almost 1.3 % which increases the number of operating firms on that link by 1.12% and let average quantities and productivity drop by about -2.87%. Still, as a result of these effects, export increase by 8.7%.

**Table 6: Example information from Melitz model by trade link**

Meltz, trade matrix [0]		Percentage diff. to Scenarios no Shock									
Exporter		Products									
Australia, New Zealand		Heavy Manufacturing									
Years		shock									
	Domestic sales	Australia, New Zealand	East Asia	Southeast Asia	South Asia	North America	Latin America	European Union 25	Middle East and North Africa	Sub-Saharan Africa	Rest of World
<b>Firm price</b>	1.00	1.00	1.01	1.01	1.03	0.99	1.01	1.00	0.99	0.98	1.00
	-0.15%	-0.09%	1.38%	0.51%	3.05%	-0.67%	1.30%	-0.37%	-0.58%	-2.35%	-0.45%
<b>Number of operating firms</b>	0.97	0.97	1.04	1.00	1.12	0.95	1.03	0.96	0.95	0.87	0.95
	-3.22%	-2.96%	3.78%	-0.24%	11.92%	-5.49%	3.41%	-4.17%	-5.12%	-12.66%	-4.55%
<b>Avg. output per firm</b>	102.25	7.52	18.54	6.78	4.54	6.60	0.60	9.33	0.15	1.53	4.20
	0.25%	0.19%	-1.26%	-0.41%	-2.87%	0.76%	-1.19%	0.68%	0.68%	2.51%	0.55%
<b>Avg. productivity per firm</b>	0.34	0.34	0.29	0.34	0.29	0.30	0.30	0.35	0.35	0.35	0.35
	0.25%	0.19%	-1.26%	-0.41%	-2.87%	0.76%	-1.19%	0.68%	0.68%	2.51%	0.55%
<b>Total output sold</b>	98.97	7.30	19.24	6.76	5.08	6.24	0.62	8.94	0.14	1.34	4.01
	-2.98%	-2.77%	2.47%	-0.65%	8.71%	-4.77%	2.18%	-3.73%	-4.48%	-10.47%	-4.03%

## 5. Conclusion

While the Armington specification based on regionally differentiated goods provides a popular and robust specification for numerical simulations of trade policy, it fails to explain empirical observations at firm-level in newer international trade literature. Recent models of international trade with heterogeneous firms overcome the limitations of the Armington specification and can at the same time be relatively easy integrated into aggregated equilibrium analysis. That has opened up the opportunity for CGE models to better depict and analyze mechanisms through which productivity and number of varieties impact the extensive margin of trade. This paper discusses an operational implementation of the firm heterogeneity theory of Melitz (2003) into the CGEBox model which addresses the shortcoming of the Armington specification while being relatively simple. It has proven numerically stable at least for medium sized aggregations of the GTAP data base.

## Appendix 1: Price linkages

The following section presents the price linkages between firms and agents. Firm’s price  $PF_{irs}$  (inclusive of domestic transport margin) is defined as the price received by producers in region  $r$  for commodity  $i$  to be shipped to the sink region. If the commodity is shipped to the domestic market, the agent (purchase) price is

$$PA_{airs} = PF_{irs} (1 + \tau_{air}^{cd} + \zeta_{air}^{cd}) \quad (28)$$

where  $\tau_{air}^c$  denote the consumption (sale) tax on each specific commodity for each agent while  $\zeta_{air}^c$  is a uniform consumption tax across commodities and/or agents.

If the commodity is shipped to the other region/country, a bilateral export subsidy or tax ( $\tau_{irs}^e$ ) is applied to the firm offer price and determines the free on board (fob) price. An additional tax  $\zeta_{irs}^e$  is also introduced into (29) representing the uniform export tax across destinations.

$$PE_{irs}^{FOB} = PM_{irs} (1 + \tau_{irs}^e + \zeta_{irs}^e) \quad (29)$$

The FOB price  $P_{irs}^{FOB}$  is augmented by the international transport margin  $t_{irs}^{tmg}$  (observed from GTAP and endogenous to the model) to establish the cif (cost –insurance and freight) price:

$$PM_{irs}^{CIF} = P_{irs}^{FOB} + t_{irs}^{tmg} \quad (30)$$

The bilateral import tax ( $\tau_{irs}^m$ ) converts the cif price into the bilateral import price, and  $\zeta_{irs}^m$  reflect a uniform tax shift across source countries

$$PM_{irs} = P_{irs}^{CIF} (1 + \tau_{irs}^m + \zeta_{irs}^m) \quad (31)$$

and finally the resulting bilateral import prices are converted to the agent prices by adding a consumption tax on imported commodities:

$$PA_{airs} = P_{irs}^{CIF} (1 + \tau_{air}^{cim} + \zeta_{air}^{cim}) \quad (32)$$

While uniform shift parameters are initially set to zero, the value of all other parameters are observed from GTAP 8 database. Potential users are strongly urged to consult Hertel (1997) and Mensbrugghe (2015).

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## Appendix 2:

### Table A.1: Sectoral aggregations

The screenshot shows a software interface for 'Sector aggregation [0]'. At the top, there are controls for 'Years shock' (a slider), 'Scenarios' (a dropdown menu set to 'Tariffs'), and 'Percentage diff. to Scenarios' (a dropdown menu set to 'no Shock'). Below these controls is a search bar. The main area contains a list of sectors, each with a title and a list of sub-sectors in parentheses:

- Grains and Crops** (Paddy rice, Wheat, Cereal grains, n.e.s., Vegetables and fruits, Oil seeds, Sugar cane and sugar beet, Plant-based fibers, Crops, n.e.s., Processed rice, )
- Livestock and Meat Products** (Bovine cattle, sheep and goats, horses, Animal products n.e.s., Raw milk, Wool, silk-worm cocoons, Bovine cattle, sheep and goat, horse meat products, Meat products n.e.s., )
- Mining and Extraction** (Forestry, Fishing, Coal, Oil, Gas, Minerals n.e.s., )
- Processed Food** (Vegetable oils and fats, Dairy products, Sugar, Food products n.e.s., Beverages and tobacco products, )
- Textiles and Clothing** (Textiles, Wearing apparel, )
- Light Manufacturing** (Leather products, Wood products, Paper products, publishing, Metal products, Motor vehicles and parts, Transport equipment n.e.s., Manufactures n.e.s., )
- Heavy Manufacturing** (Petroleum, coal products, Chemical, rubber, plastic products, Mineral products n.e.s., Ferrous metals, Metals n.e.s., Electronic equipment, Machinery and equipment n.e.s., )
- Utilities and Construction** (Electricity, Gas manufacture, distribution, Water, Construction, )
- Transport and Communication** (Trade, Transport n.e.s., Sea transport, Air transport, Communication, )
- Other Services** (Financial services n.e.s., Insurance, Business services n.e.s., Recreation and other services, Public administration and defense, education, health services, Dwellings, )

### Table A.2: Regional aggregations

The screenshot shows a software interface for 'Regional aggregation [0]'. It has the same controls as Table A.1: 'Years shock' slider, 'Scenarios' dropdown (set to 'Tariffs'), and 'Percentage diff. to Scenarios' dropdown (set to 'no Shock'). Below the search bar, the following regions are listed with their constituent countries:

- Australia, New Zealand** (Australia, New Zealand, Rest of Oceania, )
- East Asia** (China, Hong Kong, Japan, Korea, Mongolia, Taiwan, Rest of East Asia, )
- Southeast Asia** (Cambodia, Indonesia, Lao People's Democratic Republ, Malaysia, Philippines, Singapore, Thailand, Viet Nam, Rest of Southeast Asia, )
- South Asia** (Bangladesh, India, Nepal, Pakistan, Sri Lanka, Rest of South Asia, )
- North America** (Canada, United States of America, Mexico, Rest of North America, )
- Latin America** (Argentina, Bolivia, Brazil, Chile, Colombia, Ecuador, Paraguay, Peru, Uruguay, Venezuela, Rest of South America, Costa Rica, Guatemala, Honduras, Nicaragua, Panama, El Salvador, Rest of Central America, Caribbean, )
- European Union 25** (Austria, Belgium, Cyprus, Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, Ireland, Italy, Latvia, Lithuania, Luxembourg, Malta, Netherlands, Poland, Portugal, Slovakia, Slovenia, Spain, Sweden, United Kingdom, )
- Middle East and North Africa** (Rest of Western Asia, Egypt, Morocco, Tunisia, Rest of North Africa, )
- Sub-Saharan Africa** (Benin, Burkina Faso, Cameroon, Cote d'Ivoire, Ghana, Guinea, Nigeria, Senegal, Togo, Rest of Western Africa, Central Africa, South Central Africa, Ethiopia, Kenya, Madagascar, Malawi, Mauritius, Mozambique, Rwanda, Tanzania, Uganda, Zambia, Zimbabwe, Rest of Eastern Africa, Botswana, Namibia, South Africa, Rest of South African Customs, )
- Rest of World** (Oman, Israel, Switzerland, Norway, Rest of EFTA, Albania, Bulgaria, Belarus, Croatia, Romania, Russian Federation, Ukraine, Rest of Eastern Europe, Rest of Europe, Kazakhstan, Kyrgyzstan, Rest of Former Soviet Union, Armenia, Azerbaijan, Georgia, Bahrain, Iran Islamic Republic of, Kuwait, Qatar, Saudi Arabia, Turkey, United Arab Emirates, Rest of the World, )