Agricultural Sector Model BenIMPACT
(Benin Integrated Modelling System for Policy Analysis, Climate and Technology Change)

- Technical documentation -

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Model equations

UPPER CASE: VARIABLES

lower case: Parameters and indices

1.1 Resource restrictions and use, and inputs to production

The land restriction has to be met in every period. \( plevf \) denotes the percentage of crop coverage per quarter. The inequality is complementary to the shadow price of land \( VSPLD \):

Land restriction

\[
\text{landbound}_t \geq \sum_j VLEVl_{t-j} \cdot plevf_{t-j} \perp VSPLD_{t-j}
\]

The total amount of family time resources can be spent on farm labour, off-farm labour, and leisure. The inequality is complementary to the shadow price of labour \( VSPLB \):

Family labour restriction

\[
plab{i,j} \geq VLABF_{i,j} + VLABS_{i,j} + VHCON_{i,j} \perp VSPLB_{i,j}
\]

Family labour is total farm labour required minus hired labour:

Family labour = total - hired labour

\( VLABF_{i,j} = VLABT_{i,j} - VLABH_{i,j} \)

Principally, labour will be hired as long as the shadow price for farm labour is equal or greater than the going wage rate. This wage rate increases when more hired labour is demanded by the farm sector.

Condition for hiring labour

\[
VPRIS_{i,j} \sum \left( \begin{array}{c} plabr_{i,j} - plabf_{i,j} - phcon_{i,j} \\ -plab_{i,j} + plabs_{i,j} \\ \end{array} \right) \left( \begin{array}{c} -plab_{i,j} - VLABF_{i,j} - VHCON_{i,j} \\ -VLABH_{i,j} + VLABS_{i,j} \\ \end{array} \right) \leq 0 \perp VLABH_{i,j}
\]

Principally, family labour resources will be employed in off-farm labour as long as the shadow price for farm labour is equal or lower than the going wage rate. This wage rate decreases when farm households seek more employment in the non-farm-sector.

Condition for off-farm labour

\[
VSPLB_{i,j} - VPRIS_{i,j} \sum \left( \begin{array}{c} plabr_{i,j} - plabf_{i,j} - phcon_{i,j} \\ -plab_{i,j} + plabs_{i,j} \\ \end{array} \right) \left( \begin{array}{c} -plab_{i,j} - VLABF_{i,j} - VHCON_{i,j} \\ -VLABH_{i,j} + VLABS_{i,j} \\ \end{array} \right) \geq 0 \perp VLABS_{i,j}
\]

Fodder restriction (determining the quarterly limitation factor \( VRRED \) for the ruminants’ herd sizes)

\[
\text{yield}_{i,k,\text{q},t} \geq \left( \text{landbound}_t - \sum_{j} VLEVl_{t-j} \cdot plevf_{t-j} \right) + pfore_{k,j} + VLEVl_{k,j} \cdot pcres_{k,j}
\]

\[
\geq \sum_{k} \left( VLEVl_{k,j} \cdot pfodi \right) \perp VRRED_{i,j}
\]

Determine the period \( t \) with the highest limitation factor for ruminants. The expression is a non-linear approximation of the largest element of the sum.

\[
VMRED_{i} \geq \left( \sum VRRED_{i,j} \right)^{1/2}
\]

Update of forest area

\[
pfore_{i,\text{q},t+1} = pfore_{i,\text{q},t} \left( 1 - \frac{pfore_{i,\text{q},t}}{\text{landbound}_{t}} \cdot \text{poppres}_{i,\text{q},}\cdot K \right)
\]
where
\[ K = \frac{\sum (p_{fore_{i,ras}} (1 + p_{defo}))}{\sum (p_{fore_{i,ras}} p_{landbound_{i}} + p_{poppres_{i,ras}})} \]

Water requirements of livestock
\[
V\text{AW}_{\text{r,ras},i} = \sum_{r_{i,j}^{5}} V\text{FER}_U \cdot p_{awat_{r,ras},i} \cdot (4.303 + 0.906 \cdot e^{1.148 \cdot p_{defo_{r,ras},i}})
\]
\[
+ V\text{FER}_L \cdot p_{yield_{r,ras},i} \cdot 0.03 + \sum_{r_{i,j}^{5}} V\text{FER}_L \cdot p_{awat_{r,ras},i}
\]

1.2 Production output and supply

Annual crop yields are a function nitrogen application per hectare, \( V\text{FER}_L \). The function is a quadratic approximation of a series of yield simulations using the crop model EPIC.
\[
V\text{YIEL}_{i,5} = p_{yiel_{i,j}} \cdot (\text{yielbeta}_{i,5} + \text{yielbeta}_{i,5}^{2} \cdot V\text{FER}_L + \text{yielbeta}_{i,5}^{3} - V\text{FER}_L^{2})
\]

The use of fertiliser is determined with the following inequality, which is the foc of the maximisation problem w.r.t. to \( V\text{FER}_L \):
\[
\left( \text{price} + \text{SPSFT} \right) - \text{marginal costs} \geq 2 \cdot \sum \left( \text{price}_{i,j} \cdot \text{VPRIS}_{i,j} \cdot \text{plosf}_{i,j} \right) \cdot \text{yielbeta}_{i,5} + \sum \left( \text{price}_{i,j} \cdot \text{VPRIS}_{i,j} \cdot \text{plosf}_{i,j} \right) \cdot \text{yielbeta}_{i,5}^{2} - V\text{FER}_L^{2}
\]

with \( V\text{FER}_{i,j} = V\text{FER}_{i,j} \cdot p\text{ncnt} \) where \( p\text{ncnt} \) is the nitrogen content of the fertiliser applied.

The gross revenues per hectare are yield times price, corrected for the loss coefficient \( \text{plosf} \). Gross margins are gross revenues minus variable costs per hectare.

Gross revenues
\[ V\text{GREV}_{i,j} = \sum_{i,j} V\text{YIEL}_{i,j} \cdot \text{VPRIS}_{i,j} \cdot \text{plosf}_{i,j} \]

Gross margins
\[ V\text{GMAR}_{i,j} = V\text{GREV}_{i,j} - \text{VEVL}_{i,j} \cdot \text{plosf}_{i,j} \cdot \text{VPRIS}_{i,j} \]

Activity levels (in the case of BenImpact the crop areas) are determined by the first-order condition of the objective function w.r.t. activity levels. With the use of positive mathematical programming, the objective function is quadratic in activity levels, which implies that the ensuing f.o.c. still contains activity levels. The inequality contains gross revenues, costs for inputs and resources (labour, land) and the PMP coefficients \( \text{pmpa} \) (intercept) and \( \text{pmpb} \) (slope):

Crop areas
\[
\begin{align*}
\text{VEVL}_{i,j} &= \frac{\text{price}_{i,j} + \text{SPSFT}_{i,j}}{\text{plosf}_{i,j} \cdot \text{VPRIS}_{i,j} \cdot \text{VEVL}_{i,j} + \text{VSPFD}_{i,j}} \\
\text{VEVL}_{i,j} &= \frac{\text{price}_{i,j} + \text{SPSFT}_{i,j}}{\text{plosf}_{i,j} \cdot \text{VPRIS}_{i,j} \cdot \text{VEVL}_{i,j} + \text{VSPFD}_{i,j}}
\end{align*}
\]

Livestock herd sizes are determined by a simple price response function applied to a base herd size \( \text{plevl} \). The fodder limitation factor \( \text{VMRED} \) is applied for ruminants.
Extensive livestock herd size

\[ V_{LEVI_{ij}} = \left( \sum_{j} V_{PRES_{ij}} \right) \frac{1}{4} \cdot \frac{1}{p_{agro_{ij}}} \cdot \{V_{LEVI_{ij}} + VMRED_{ij} + V_{LEVI_{ij}} \cdot (1-VMRED_{ij}) \} \]

The variable denoting the response of herd size to price is a non-linear function of the price changes and a constant elasticity \( p_{asel} \): 

Price response of livestock

\[ V_{PRES_{ij}} = \left( \frac{V_{PRIS_{ij}}}{p_{prpi_{ij}}} \right)^{\alpha_{ij}} \cdot \{ j \rightarrow j(i) \} \]

Intensively managed livestock herd size

\[ 0 \geq V_{REV_{i,am}} - \left( \pi_{aco_{i,am}} + p_{lab_{i,am}} \cdot V_{SPLB_{ij}} + p_{lev_{i,am}} \cdot V_{SPLD_{ij}} + p_{ife_{i,am}} \cdot V_{PRIS_{ij}} \right) \]

Gross production on farm is activity levels (crop areas, livestock numbers) multiplied by the yield per unit of the activity:

Gross production on farm

\[ V_{PROD_{ij}} = V_{LEVI_{ij}} \cdot V_{YIEL_{ij}} \cdot \{ j \rightarrow j(i) \} \]

 Marketable production is gross production times the loss coefficient. For neighbouring countries, marketable production is determined by simple linear supply functions with the coefficients \( q_{pia} \) (intercept) and \( q_{pib} \) (slope):

Production entering markets

\[ V_{MAPR_{ij}} = V_{PROD_{ij}} \cdot p_{losf_{ij}} \cdot \{ V_{PRIS_{ij}} - q_{pia_{ij}} \} / q_{pib_{ij}} \]

Processing must not exceed quarterly capacities \( p_{fact} \) (relevant for cotton). The constraint is complementary to the shadow price of processing capacity:

Processing capacities

\[ p_{fact_{ij}} \geq V_{PROC_{ij}} \geq V_{SPPC_{ij}} \]

If processing capacities are not fully used, producers receive the full guaranteed price \( prpri \). If capacities are at their limits, producer prices will be reduced by the shadow price. Processing levels are complementary to this price relation:

Processing profitability

\[ V_{PRIS_{ij}} \geq prpri_{ij} - V_{SPPC_{ij}} \geq V_{PROC_{ij}} \]

The market balance contains supply (marketable production, net stock releases, and import flows) and demand elements (consumption, processing, and export flows). The equation is complementary to the market price \( V_{PRIS} \):

Market clearing condition

\[ V_{MAPR_{ij}} \geq \left( \sum_{i} V_{STIN_{i,am}} - V_{STIN_{ij}} \right) - \sum_{i} V_{TRAN_{ij}} \geq VHCON_{ij} + V_{PROC_{ij}} - \sum_{i} V_{TRAN_{ij}} \geq V_{PRIS_{ij}} \]

Transport flows \( V_{TRAN} \) will occur when interregional price differences are greater or equal to transport costs:

Spatial price transmission

\[ V_{PRIS_{ij}} \geq t_{i,j} \cdot V_{PREF_{ij}} \geq V_{PRIS_{ij}} \geq V_{TRAN_{ij}} \]

Storage \( V_{STIN} \) will be profitable when intertemporal price differences are greater or equal to storage costs:
Temporal price transmission \[ VPRIS_{r,t} + sc_{r,ppp} \geq VPRIS_{r,t+1} - VSTIN_{r,t} \]

1.3 Income of rural households

So far, income from production is equal to the sum of gross margins per farming activity minus expenditure for hired labour.

Income from production minus labour costs

\[ VINC_{r} = \left( \sum_{i} VGMAR_{r,i} \cdot VLEVL_{r,i} - \sum_{i} VLABH_{r,i} \cdot VSPLB_{r,i} \right) \]

Off-farm income consists of wage income from off-farm labour and a residual component \( p\text{incp} \):

Income from off-farm activities

\[ VINCR_{r} = \sum_{i} VLBS_{r,i} \cdot VSPLB_{r,i} + p\text{incp}_{r} \]

Total income per capita

\[ VINCC_{r} = (VINC_{r} + VINCR_{r})/(ppopr \cdot ppopt) \]

As leisure is among the consumption goods, an income equivalent from ‘leisure consumption’ has to be added to the income used in the GL demand system to maintain consistency:

Income equivalent from leisure per capita

\[ VINCL_{r} = \left( \sum_{i} HCON_{r,i} \cdot VSPLB_{r,i} \right)/(ppopr \cdot ppopt) \]

1.4 Commodity demand and consumption

As only rural households are considered, the consumer price is assumed to be equal to the producer price. The consumer price for leisure is the shadow price for labour:

Supply price determines consumer price

\[ VPRID_{r,i} = VPRIS_{r,i} \quad (\text{if } i \text{ is a consumer good}) \]
\[ VSPLB_{r,i} \quad (\text{if } i \text{ is leisure}) \]

Consumption per capita is determined in a Generalised Leontief system of demand. Both relative prices of commodities \( VPRID \) and rural incomes (\( VINCC \) and \( VINCL \)) drive the results.

Rural consumer demand per capita

\[ VHCRP_{r,i} = \frac{GIS_{r,i}}{GS_{r} \cdot (VINCC_{r} + VINCL_{r} - FS_{r})} + ds_{r,i} \]

With

\[ GIS_{r,i} = \sum_{j} bs_{r,i,j} \cdot \sqrt{VPRID_{r,j} + VPRID_{r,j} \cdot VPRID_{r,j}} \]
\[ GS_{r} = \sum_{j} bs_{r,i,j} \cdot \sqrt{VPRID_{r,j} + VPRID_{r,j} \cdot VPRID_{r,j}} \]
\[ FS_{r} = \sum_{j} VPRID_{r,j} \cdot ds_{r,j} \]

The remaining two equations determine the total demand of both rural and urban households in absolute terms and per capita. Urban consumption (\( p\text{conu} \)) is kept fix during a simulation and is shifted according to population growth between simulation years.

Total consumer demand for commodities

\[ VHCON_{r,i} = VHCRP_{r,i} \cdot ppopr \cdot ppopt / 1000 + p\text{conu}_{r,i} \]

Total consumer demand per capita

\[ VHCP_{r,i} = VHCON_{r,i} / ppopr \cdot 1000 \]
### List of abbreviations

<table>
<thead>
<tr>
<th>Variables</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>VAWAT(r,sim,t)</td>
<td>Total water requirements for all productive livestock</td>
<td>1000 tons</td>
</tr>
<tr>
<td>VGREV(r,j)</td>
<td>Gross revenue per hectare or livestock unit</td>
<td>1000 FCFA</td>
</tr>
<tr>
<td>VGMAR(r,j)</td>
<td>Gross margin per hectare or livestock unit</td>
<td>1000 FCFA</td>
</tr>
<tr>
<td>VHCON(r,i,t)</td>
<td>Consumption of products</td>
<td>1000 tons</td>
</tr>
<tr>
<td>VHCP(t)</td>
<td>Per capita consumption</td>
<td>kg per capita</td>
</tr>
<tr>
<td>VIFED(r,i,t)</td>
<td>Feed demand intensive livestock keeping</td>
<td>1000 tons</td>
</tr>
<tr>
<td>VINCC(r)</td>
<td>Total income per capita</td>
<td>1000 FCFA per capita</td>
</tr>
<tr>
<td>VINEL(r,j)</td>
<td>Levels of hypothetic activities</td>
<td>1000 heads</td>
</tr>
<tr>
<td>VINCR(r)</td>
<td>Income from off-farm activities</td>
<td>1000 FCFA per capita</td>
</tr>
<tr>
<td>VLABT(r,t)</td>
<td>Total labour in agricultural production</td>
<td>mio hours</td>
</tr>
<tr>
<td>VLABH(r,t)</td>
<td>Labour hired</td>
<td>mio hours</td>
</tr>
<tr>
<td>VLABS(r,t)</td>
<td>Family labour sold</td>
<td>mio hours</td>
</tr>
<tr>
<td>VLEVL(r,j)</td>
<td>Activity level</td>
<td>1000 ha or 1000 heads</td>
</tr>
<tr>
<td>VMAPR(r,i,t)</td>
<td>Marketable production</td>
<td>1000 tons</td>
</tr>
<tr>
<td>VMRED(r)</td>
<td>Maximum value of VRRED over all periods</td>
<td>percent</td>
</tr>
<tr>
<td>VPRIS(r,i,t)</td>
<td>Supply responding to VPRIS in livestock husbandry</td>
<td>percent</td>
</tr>
<tr>
<td>VPROD(r,i,t)</td>
<td>Gross production or use</td>
<td>1000 tons</td>
</tr>
<tr>
<td>VSPBL(r,t)</td>
<td>Wage rate (shadow price for labour)</td>
<td>FACFA per hour</td>
</tr>
<tr>
<td>VSPRD(r,t)</td>
<td>Land rent (shadow price for land)</td>
<td>1000 FCFA per ha</td>
</tr>
<tr>
<td>VSPPC(r,i,t)</td>
<td>Shadow price for processing capacity</td>
<td>1000 FCFA per ton</td>
</tr>
<tr>
<td>VSTIN(r,i,t)</td>
<td>Stock in</td>
<td>1000 tons</td>
</tr>
<tr>
<td>VYIEL(r,i,j,t)</td>
<td>Crop yields and yields of livestock</td>
<td>1000 tons</td>
</tr>
</tbody>
</table>

### Constants

<table>
<thead>
<tr>
<th>Constants</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>bs(r,i,j)</td>
<td>Terms in front of square roots in GL Expenditure function</td>
</tr>
<tr>
<td>ds(r,i,t)</td>
<td>Constant terms in GL Expenditure function</td>
</tr>
<tr>
<td>K</td>
<td>Constant term for updating the regional forest area</td>
</tr>
<tr>
<td>landbound(r)</td>
<td>Upper bound on land use</td>
</tr>
<tr>
<td>pafor(r)</td>
<td>Regional forest areas</td>
</tr>
<tr>
<td>pagro(j)</td>
<td>Yearly growth rate of animal numbers</td>
</tr>
<tr>
<td>pasel(r,j)</td>
<td>Supply elasticity in livestock keeping</td>
</tr>
<tr>
<td>pawat(r,j,t)</td>
<td>Water requirements of livestock</td>
</tr>
<tr>
<td>pconu(r,i,t)</td>
<td>Urban consumption</td>
</tr>
<tr>
<td>pcres(r,j,t)</td>
<td>Crop residues useable for livestock feeding</td>
</tr>
<tr>
<td>pdeo('ben')</td>
<td>Global deforestation rate in Benin</td>
</tr>
<tr>
<td>pfact(r,i)</td>
<td>Processing capacity</td>
</tr>
<tr>
<td>pfodi(jrum,t)</td>
<td>Feed requirements of ruminants</td>
</tr>
<tr>
<td>pfore(r)</td>
<td>Regional forest areas</td>
</tr>
<tr>
<td>plcon(r,i,t)</td>
<td>Base consumption</td>
</tr>
<tr>
<td>piaco(r,iani)</td>
<td>Input costs for intensive livestock keeping</td>
</tr>
<tr>
<td>pifed(imon,i,t)</td>
<td>Feed requirements for monogastric animals</td>
</tr>
<tr>
<td>pinpu(r,i,j,t)</td>
<td>Quantity of input factors</td>
</tr>
<tr>
<td>plaba(r,j,i,t)</td>
<td>Labour used per activity</td>
</tr>
</tbody>
</table>
plabf(r,t)  Family labour in cropping mio hours
plabh(r,t)  Labour hired mio hours
plabi(r,t)  Labour pool of farm households mio hours
plabr(r,t)  Total regional labour pool mio hours
plabs(r,t)  Family labour sold mio hours
plevf(r,j,t)  Area requirements per activity and period percent
plevl(r,j,t)  Activity levels thousand units (ha, heads)
plosf(r,i,t)  Production losses percent
plevl(r,j,t)  Activity levels thousand units (ha, heads)
pmpa(r,j,a)  PMP term intercept
pmpb(r,j,j)  PMP term slope
pmpres(r)  Population pressure per region percent
pmpres(r)  Population pressure per region percent
ppopt(r)  Population in simulation region 1000
ppopt(r)  Population in simulation region 1000
ppotf(r,i,t)  Supply price 1000 FCFA per ton
ppotf(r,i,t)  Supply price 1000 FCFA per ton
ptfms(r,sim,t)  Regional temperature °C
pyiel(r,i,j,t)  Crop yields per hectare and yields of livestock unit tons
qpija(r,i,t)  Quadratic Profit function - International linear
qpijb(r,i,t)  Quadratic Profit function - International quadratic
sc(r,pgrp)  Storage cost derived from losses 1000 FCFA per ton
tc(r,r,pgrp)  Trade cost between regions 1000 FCFA per ton
yield(r,bio,sim,t)  Biomass of pasture and forests in dry mass tons

Indices

r, s  Regions
j  Activity
i  Product
t  Time - period (quarter)
labo  Labour
leis  Leisure
bio  Biomass
iani  Intensive livestock production
imon  Intensively kept monogastric animals
irum  Intensively kept ruminants
jc  Crop activity
jl  Livestock activity
jr um  Activity of keeping of ruminants
jcatt  Activity of keeping cattle
jmon  Activity of keeping of monogastric animals
pgrp  Production groups
rdom  Region in Benin
nc  Neighbouring countries
sim  Simulation year