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Methodological description of the CGERegEU

DRAFT VERSION

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Introduction

This paper describes a methodology behind the CGERegEU model. The general features of the model were presented in the Deliverable 3.2.1 and here we focus more on details, such as theories, equations, solutions, etc. CGERegEU layer is a system of single-country, multi-regional (up to NUTS 2 level) and multi-sectoral (above 10 sectors) computable general equilibrium models of all EU countries (27) and candidate countries. Therefore on one hand, it has to be powerful to cope with such a large number of regions and sectors in order to produce solutions for all of them. On the other hand, it has to be simple and fairly standardised to suit all of them. For these reasons it is reasonably demanding in terms of the data, so that all information is possible to collect at regional level for all the countries in question. This paper describes the main blocks of equations behind the model but tries to do this in not a very technical way.

The paper is structured as follows. In the introduction we shortly present other applied regional CGE models, which leads us to the discussion on different modelling approaches i.e.: 1) modelling in level forms versus 2) modelling through linearized representations. Our model is rooted in the latter tradition and we explain what it means. Then we discuss the structure of the model together with assumptions behind: production, consumption, public sector, investments, foreign and local trade, equilibrium conditions, etc. Last, but not least, we discuss rules for aggregating the regional result to the national levels (so called "Macros"). The last part concludes with the most important features of the model, which made it a good prototype for pan-European model.

Regional CGE modelling¹

CGE modelling has gained much attention during the last thirty years. Partridge and Rickman (1998) give the most comprehensive overview of regional and multi-regional CGE models to date. Partridge and Rickman (1998)² mentions five surveys for mostly national models on taxation policy, international trade, the development policy of less developed countries, energy, and environmental issues in their critical analysis. They also list the key features of some 28 regional CGE models in their table 1, of which over a dozen are multi-regional. There are also several sub-national models. The work of Despotakis and Fisher (1988) is an early example and a clear presentation of a CGE model for the Californian economy comparing the use of generalized (flexible) and ordinary (fixed) Leontief production functions.

Partridge and Rickman (1998)³ has a reference to Kraybill (1993)³, who himself mentions a handful of early multi-regional models and gives a short survey, although his emphasis is on comparing CGE with input–output models. An early example is Jones and Whalley (1989) who constructed a static model for six Canadian regions. In addition, Liew (1984) developed a Leontief-type multi-region model for Australia.

¹ The text, which is partly updated, is from Törmä, 2008; Rutherford and Törmä, 2009.

² Read footnote 1 in Partridge and Rickman (1998)

³ Read footnote 2 in Partridge and Rickman (1998)

Today there are several multi-regional CGE models. Just recently in 2003, the Review of Urban & Regional Development Studies published three regional CGE model articles. The research topic of Schwarm and Cutler (2003) was small city and town social accounting matrices (SAMs); Giesecke and Madden (2003) presented a large-scale dynamic multi-regional model; and Diao et al. (2003) investigated China's integration into the World Trade Organization (WTO) using a CGE model.

The CoPS modelling team of Monash University, Melbourne, Australia, has developed several regional CGE models. The MONASH model (Dixon and Rimmer, 2002; Horridge et al., 1999), which was developed from the previous ORANI model, has 56 regions, MMRF has 8 regions (Peter, Horridge, Meagher et al., 1996) and TERM model has 57 regions built up in a bottom-up manner (Wittwer and Horridge, 2007; Horridge and Wittwer, 2008; Wittwer and Horridge, 2010). TERM was replicated by now into seven countries: Brazil, China, Finland, Indonesia, South Africa, Poland, and Japan. These three types of dynamic regional CGE models have been used in a multitude of applications of Australian national and regional policies. CGE models are usually multi-purpose in their design, so they can be used flexibly in many applications. One good example is Madden (2002), who quantified the economic impact of the Sydney 2000 Olympics. Other well-known regional CGE models are the Scottish two-region AMOSRUK model (Ferguson et al., 2004) and the Brazilian 27-region B-Maria model (Haddad, 2004).

General equilibrium analysis and CGE models have gained strong interest among Finnish economists. Törmä (2005a) reviews this modelling tradition in Finland and finds out that the CGE applications included research on: EU integration (Törmä and Rutherford, 1993; Törmä et al., 1995; Vaittinen, 2004), energy and the environment (Honkatukia et al., 2003), regional policy (Kinnunen, 2005; Rutherford and Törmä, 2006) and the pension system (Alho et al., 2005). There have also been many studies relating to infrastructure development projects, such as factories, roads, railways, tunnels and mines (Törmä and Rutherford, 2002; Honkatukia et al., 2004, 2006; Törmä and Zawalinska, 2007a, b; Törmä and Reini, 2009; Reini and Törmä, 2010) and investment in culture (Törmä, 2005b). The prototype of the model applied in this project, named RegFin, also originates from this school but has many features of Australian models as well.

We can conclude that although there are several multi-regional CGE models developed, not so many of them are working for European countries. Secondly, if they exist usually they are replicated to one or a few countries, hence there has not been a pan-European regional CGE model layer working so far. Our work has a pioneer character because RegFin is replicated to over 27 countries.

Level and linearization schools

In order to understand the CGE model one needs to know which convention for writing its equations was adopted by the modeller. Generally, there are two schools in CGE modelling: 1) to specify the model in the level form or 2) to use linearized representations for its equations. The decision is up to the modeller but it is also determined to some

extent by the software used. Most of the CGE models written in GAMS are in the level forms while those in GEMPACK are linearized⁴. The variables in the linearized equations are usually interpreted as percentage changes in the original variables. Our model is from the latter tradition, so it is important to understand the difference in its notation and interpretation of the results comparing to the level form fashion. Besides, we also discuss how an accurate solution can be found for a linearized model and which algorithms we can use.

Linearization of equations from the level form is done mathematically by using simple conversion rules. Some of them are presented below⁵ in Table 1, where level variables are denoted with capital letters (X, Y, A) and percentage change variables with lower case letters (x, y, a, p).

Table 1 Conversion rules between the levels and percentage change forms

<u>levels form</u>	<u>percentage change form</u>
X	$x = 100 \Delta X / X$
3X	x
XY	$x + y$
X/Y	$x - y$
X^a	ax
X + Y	$(Xx + Yy) / (X + Y) = S_x x + S_y y,$ $S_x = X / (X + Y), S_y = 1 - S_x$

The examples of transforming real equations from the level into linear form would be as follows:

Eq. 1 $D = P Q$

The equation says that value level of the commodity (D) is equal to its quantity level Q (in tons) at certain price level P (in currency). The linearised version of it would be:

Eq. 2 $d = p + q$

The equation says that percentage change in the value of the commodity (d) equals to the percentage change in its quantity (q) plus percentage change in its price (p).

⁴ Actually GEMPACK has a useful feature, that one can enter an equation in a level form, and the software automatically converts it into the percentage change form.

⁵ For more examples, see Annex 1.

Another example would be:

$$\text{Eq. 3} \quad X = \text{CES}(X_i/A_i),$$

$$\text{then } x_i - a_i = x - \sigma (p_i + a_i - p), \text{ and } p = \sum S_i (p_i + a_i)$$

CES is a Constant Elasticity of Substitution production function, X is output, X_i is primary production input or factor; A is an index for technological change and σ is substitution elasticity, and p is a primary factor or a weighted production price.

So in CGERegEU model all the main behavioural equations are expressed in linearized forms, like primary factor demands presented above (see also Horridge, 2003 pp 65-66). However, accounting conditions, such as commodity and factor market clearance conditions, are expressed in terms of levels, as it is more convenient that way.

The linearization requires specific solution methods. Hertel et al. (1992, 395-396) show how a linearized representation can be used to obtain accurate solutions. They illustrate this in a general case where there is just one exogenous variable x and one endogenous variable y , and where the only equation of the model $g(x, y) = 0$ is as shown below.

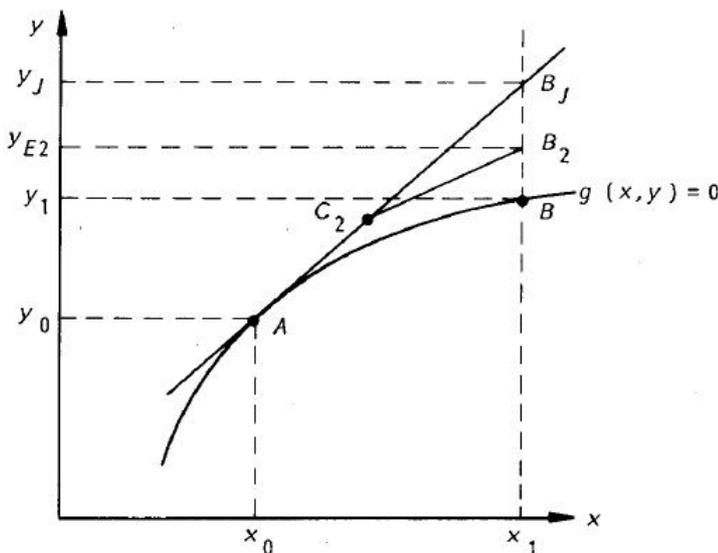


Figure 1 Illustration of Euler and Johansen methods

Suppose we know an initial solution of the model, say $y = y_0$ when $x = x_0$ (point A) and we wish to calculate the value of y when x increases to x_1 . The true solution is $y = y_1$ (point B). In this case the linearized representation of the equation is used to calculate the gradient dy/dx (in terms of x and y) at any point on the curve $g(x, y) = 0$ (and at points just off it).

Source: Hertel et al. (1992, 395-396)

Johansen's method is to calculate dy/dx at A and then, in passing from x_0 to x_1 , to move along the tangent to the curve at A. This brings us to the point B_j in the figure and so this produces the estimate y_j . The idea behind other more accurate methods is to follow the curve $g(x, y) = 0$ more closely. For example, start out in the direction of the tangent at A but, after a small distance, stop, recompute the direction to move in (or dy/dx in terms of the new values of x and y reached), move in the new direction for a short distance, then recompute the direction, and so on until x reaches x_1 .

Euler's method is to divide the interval $[x_0, x_1]$ into N equal subintervals and then proceed as indicated above, recomputing the direction to move in at the end of such subinterval. This is shown in Figure 1 for $N = 2$, when the path is from A to C_2 to B_2 which produces the

approximate solution y_{E2} . The important point to note is that, by taking N sufficiently large, the approximate solution y_{EN} obtained can be made arbitrarily close to the true solution y_1 . Johansen's method produces the Euler result y_{E1} when $N = 1$.

It is clear from the discussion above that Johansen's method is very rough, and Euler's method is superior. GEMPACK software offers four solution methods, so apart from the two described above, the two others are the Gragg and Midpoint methods.

To conclude, we presented two approaches to writing CGE model equations (in level and linearize forms) and both are equally popular. We however follow the second convention determined by the GEMPACK adjusted to it. We showed also that there are powerful solution methods, so obtaining accurate solutions is possible. What is more, for the same model and data, the two approaches have a high probability for giving equivalent solutions. Evidence for this is the results presented in Hertel et al (1992, 398-399).

Model structure

The structure described below explains the main features of the static multi-regional, multi-sectoral CGE single-country models that constitute the new layer for the CAPRI model. The specific equations and assumptions are presented for economic behaviour of the economy actors such as producers, consumers and public sector. We use the level and percentage change form notations for value-added in the production block in order to understand how the forms are linked. Then we continue with the percentage change forms as it is originally in the model code. We explain the notations and use real variable names⁶.

Production

Our model is based on typical neoclassical or Walrasian⁷ assumptions: we assume that producers face perfect competition, they are price takers at the markets which all clear out not allowing for any extra profits (zero pure profit assumed). Producers also face constant returns to scale. They are operating in a small open economy, so foreign prices are exogenous.

In perfectly competitive markets no participant has market power. This means that producers or firms are sufficiently small, and cannot therefore influence the prices which are determined at commodity, and factor markets, so the firms take them as given. All markets (except the labour market) are assumed to clear out, so prices will adjust until there is equilibrium where demand and supply are equal. General equilibrium demands that all markets are in equilibrium at the same time.

⁶ Only sometimes when data is read directly from the SAM for convenience, we have to use equivalents to variable names.

⁷ Note that Walras assumes all markets to be in equilibrium, however, we allow for unemployment in the labour market.

Entry and exit of firms is free in competitive markets, so positive (negative) profits tempt more (less) firms to enter (exit) the market. This restricts firms to earn excess or pure profits. However they obtain income from sales, which is sufficient to pay the capital and land rents, as well as wages, so cover their production costs.

The production technology operates under constant returns to scale. This means that whenever there is a change in the level of the primary factors' use, the production will change by the same proportion. Finally, the country is assumed to be sufficiently small in the world economy, so it cannot influence the world market prices but takes them as given. This means that both export and import prices are exogenous.

The assumptions in our model originate from neoclassical economic theory and are typical for most CGE models. The model has solid microeconomic foundations, but theoretically there is a scope for its improvement, e.g. by relaxing some of the assumptions, as for example changing into imperfect competition. However, it would cause many complications because we would need to decide on sectors in each country (or even region) which have imperfect competition, and also we would need to decide on the character of the imperfections. There are many alternative imperfect market forms to choose from. Adding new features for the model of this size would require gathering lots of new information, which is very much time consuming and costly. It is quite probable that for such a large application the costs would exceed the benefits. So we believe it is not within the scope of this study. However, certainly adding new features (including dynamics, relaxing assumptions, etc.) open the opportunity for the continuation of the project.

Multi-level Armington specification of production choices

The producers are assumed to be profit maximizers. They choose an optimal mix of the primary and other factors. This is described as a multi-stage decision problem, where separability assumption breaks the industry's problem into a sequence of nests. This assumption is a standard modelling practice and has practical implication of reducing significantly the number of estimated parameters required in implementing the model (Honkatukia and Vaitinen, 2003). In the model there are six nests as depicted in Figure 2. We first describe them in general and after that we precede into more details and particular equations.

In the first nest, the producer chooses between other physical capital and land subject to the constant elasticity of substitution (CES) technology. We call the output aggregate capital. In the second nest, value-added is a CES function of aggregate capital and labour. In the third nest, firms combine value added and intermediate commodities with fixed coefficients via Leontief function (no substitutability between factors) in order to form a local output. In the fourth nest, domestic commodities are combined via CES function from local output and domestic imports. Domestic commodities and goods from the rest-of-the-world (ROW) create, via CES function, the so called Armington composite. In the nest six this composite is split via constant elasticity of transformation (CET) function into three

parts: i) commodities used locally (Armington goods), ii) commodities produced for domestic exports, and iii) commodities produced for foreign exports.

Figure 2 Nested structure of production

Nest 6: ACT is split via CET into local goods, domestic exports and foreign exports.

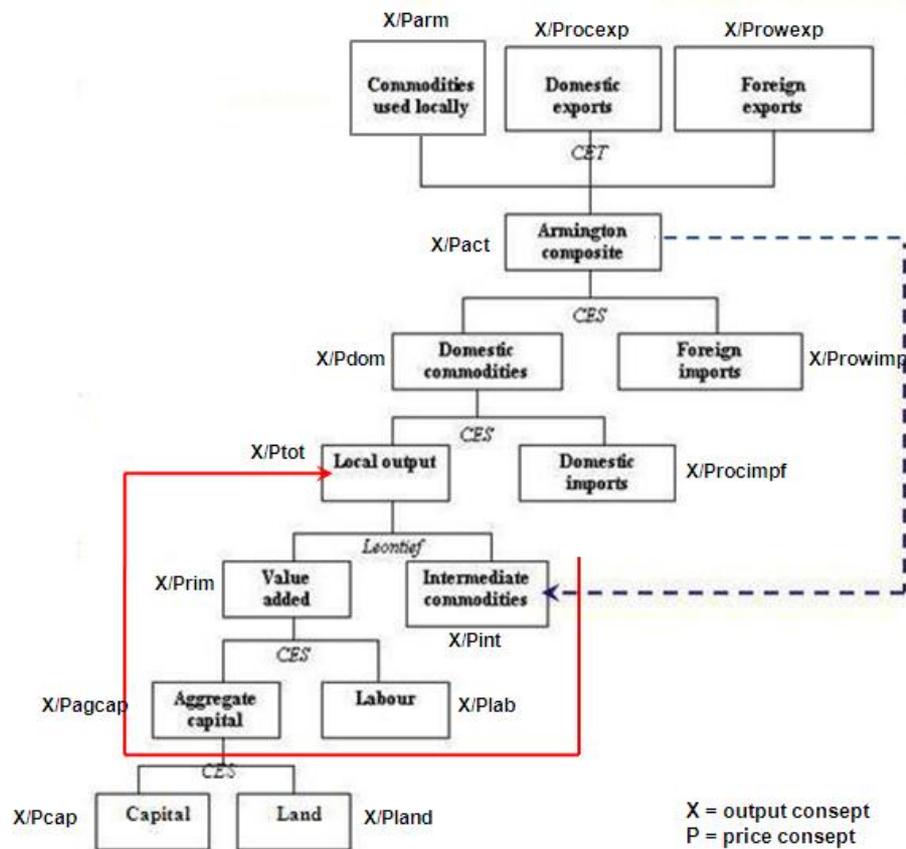
Nest 5: Armington composite is a CES function of domestic commodities and foreign imports

Nest 4: Domestic commodities are a CES function of local output and domestic imports.

Nest 3: Local output is a Leontief function of VA and Intermediate commodities.

Nest 2: Value Added is a CES function of aggregate capital and labor.

Nest 1: Aggregate capital is a CES function of land and other physical capital



It is important to realize how the model handles the substitution between locally-made, rest-of-the-country (ROC) and rest-of-the-world commodities (ROW) mentioned above. This is implemented through so called “Armington assumption” that states domestic and foreign commodities are qualitatively different (Armington, 1969). This explains cross-hauling in trade: i.e. European food is exported abroad, but at the same time foreign food is imported to Europe. This means that domestic and foreign commodities are not perfect substitutes, so the substitution elasticity must be less than infinity. The same principle can be used for domestic imports and exports to/from outside the local region. This affects the producers’ choices, which are organized in the nested structure. Hence the nests from 4 to 6 are usually called the Armington nests.

In the following chapter we explain the core equations of the producers’ behaviour using level notation for value added and then continuing with the percentage form representation.

Profit maximization problem of the producer

Let's consider the long run situation where both primary factors are flexible. The problem in its general form for value added, and by using the notation of the model⁸, is as follows:

$$\begin{aligned} & \text{Max } (X_{\text{lab}(i,r)}, X_{\text{agcap}(i,r)}) \\ & \text{subject to } P_{\text{prim}(i,r)} f(X_{\text{lab}(i,r)}, X_{\text{agcap}(i,r)}) - P_{\text{labpt}(i,r)} X_{\text{lab}(i,r)} - P_{\text{agcappt}(i,r)} X_{\text{agcap}(i,r)} \end{aligned}$$

$X_{\text{lab}(i,r)}$, $X_{\text{agcap}(i,r)}$ are respectively quantity of labour and aggregate capital demanded

$P_{\text{prim}(i,r)}$ is price of value added

f is a production function, leading to $X_{\text{prim}(i,r)}$ (quantity of value added)

$P_{\text{labpt}(i,r)}$, $P_{\text{agcappt}(i,r)}$ are respectively labour wage and aggregate capital rent cost to firm

On the left hand side are the revenues from sales, and on the right side the production costs. The difference between the two is a profit. The condition for the optimal choice of the factors is:

$$\begin{aligned} P_{\text{prim}(i,r)} \text{MP}_{\text{lab}(i,r)}(X_{\text{lab}(i,r)^*}, X_{\text{agcap}(i,r)^*}) &= P_{\text{labpt}(i,r)} \\ P_{\text{prim}(i,r)} \text{MP}_{\text{agcap}(i,r)}(X_{\text{lab}(i,r)^*}, X_{\text{agcap}(i,r)^*}) &= P_{\text{agcappt}(i,r)} \end{aligned}$$

Where:

MP is a marginal product of the factor (extra output per one unit increase in use of factor)

* indicates profit maximizing choice of the factor

The firm has made the optimal choice of factors, if the value of the marginal product (left hand side) of each factor is equal to its price. At the optimal choice, the firm's profits cannot increase by changing the level of any input.

We have a CES production function in value-added represented as follows:

$$X_{\text{prim}(i,r)} = (\bar{\delta}_{\text{lab}(i,r)} X_{\text{lab}(i,r)}^{-\sigma(i)} + \bar{\delta}_{\text{agcap}(i,r)} X_{\text{agcap}(i,r)}^{-\sigma(i)})^{-1/\sigma(i)}$$

$\bar{\delta}$ is a factor income distribution parameter, $0 < \bar{\delta} < 1$

σ is a substitution parameter, $\sigma < 1$, $\neq 0$

$1 / (1 + \sigma)$ is a substitution elasticity between labour and aggregate capital = SIGMAPRIM

We use the profit maximizing conditions to derive the corresponding factor demand functions in the level form:

$$X_{\text{lab}(i,r)} = \bar{\delta}_{\text{lab}(i,r)}^{1/(\text{SIGMAPRIM}(i)+1)} X_{\text{prim}(i,r)} (P_{\text{labpt}(i,r)} / P_{\text{prim}(i,r)})^{-\text{SIGMAPRIM}(i)}$$

⁸ Variables beginning with a capital letter refer to levels, i refer to industries, r refers to regions, c refers to commodities. In case where c and i are used together, i refers to the column and c to the row sectors.

$$X_{\text{agcap}(i,r)} = \bar{\delta}_{\text{agcap}(i,r)}^{1/(\text{SIGMAPRIM}(i)+1)} X_{\text{prim}(i,r)} (P_{\text{agcappt}(i,r)} / P_{\text{prim}(i,r)})^{-\text{SIGMAPRIM}(i)}$$

Finally, we use the conversion rules to get to the percentage change form⁹ adding also factor-augmenting technical change ($a_{\text{lab}}, a_{\text{agcap}}$)¹⁰:

$$X_{\text{lab}(i,r)} - a_{\text{lab}(i,r)} = X_{\text{prim}(i,r)} - \text{SIGMAPRIM}(i) (p_{\text{labpt}(i,r)} + a_{\text{lab}(i,r)} - p_{\text{prim}(i,r)})$$

$$X_{\text{agcap}(i,r)} - a_{\text{agcap}(i,r)} = X_{\text{prim}(i,r)} - \text{SIGMAPRIM}(i) (p_{\text{agcappt}(i,r)} + a_{\text{agcap}(i,r)} - p_{\text{prim}(i,r)})$$

$a_{\text{lab}(i,r)}$ is a labour-augmenting technical change

$a_{\text{agcap}(i,r)}$ is an aggregate capital-augmenting technical change

We can see that demand is a function of an upper level aggregate, substitution elasticity and the price ratio or real price. This is analogical for all the nests in the production structure.

The change of the value added determines the size of the output effect. The effect from technical change will be negative (positive) if more (less) input per one unit of output is needed. The price effect will be negative (positive) if the factor price increases (decreases) more (less) than the output price. The value of the substitution elasticity will determine the size of the price effect. Finally, the relative factor prices have an effect for the allocation of factor demands. There is a sub-program in GEMPACK that can be used to calculate the contributions of aforementioned elements in the total effect.

One advantage of the percentage change form is that there are only primary parameters or elasticities in the equations. Secondary parameters, like the distribution parameters are dropped out.

In our specification of production technology, value-added is used proportional to local output:

$$X_{\text{prim}(i,r)} = \Phi_{\text{tot}(i,r)} X_{\text{tot}(i,r)}, \text{ and after adding technical change:}$$

$$X_{\text{prim}(i,r)} = X_{\text{tot}(i,r)} + a_{\text{tot}(i,r)} + a_{\text{prim}(i,r)}$$

Where:

$\Phi_{\text{tot}(i,r)}$ is an input-output coefficient

$X_{\text{xtot}(i,r)}, X_{\text{tot}(i,r)}$ is a local output

$a_{\text{tot}(i,r)}$ is all-input-augmenting technical change

$a_{\text{prim}(i,r)}$ is a primary-input-augmenting technical change

⁹ Variables beginning with a lower case letter refer to percentage change.

¹⁰ For proof, see Horridge 2003, p. 65-66. All technical changes are of the form “input-per-output”, so negative is good, like $a_{\text{lab}} = -1$ means labour saving technical change.

We have now defined value-added or the so called composite primary factor. In order to complete our description of the second and third nest of the production structure, we have to define the demand for intermediate commodities as a fixed coefficient Leontief function:

$X_{\text{int}(i,r)} = \Phi_{\text{int}(i,r)} X_{\text{tot}(i,r)}$, and after adding technical change:

$$X_{\text{int}(i,r)} = a_{\text{tot}(i,r)} + X_{\text{tot}(i,r)}$$

$X_{\text{int}(i,r)}$ is a demand for intermediate commodities

Whenever local output increases (decreases), the demand for value-added and the intermediate commodities will increase (decrease) with the same proportion, depending, however, on what is assumed to happen to the technical change.

The Armington part of the production structure begins in the nest four when the producer decides about the optimal mix of local output and domestic imports. The condition for the profit maximization is the same as before. The demand functions for local output is of the form¹¹:

$$X_{\text{tot}(i,r)} = X_{\text{dom}(i,r)} - \text{SIGMADOM}_{(i)} (p_{\text{tot}(i,r)} - p_{\text{dom}(i,r)})$$

$X_{\text{dom}(i,r)}$ is a domestic commodities aggregate

$\text{SIGMADOM}_{(i)}$ is a CES substitution elasticity between local and domestically imported commodities

$p_{\text{tot}(i,r)}$ is a price of local output

$p_{\text{dom}(i,r)}$ is a price of domestic commodities

Local demand can increase (decrease) if domestic aggregate increases (decreases) and/or the price ratio changes in favour of local production. The outcome depends also on the substitution possibilities.

The demand for domestic imports is given by the following equation:

$$X_{\text{domimpf}(i,r)} = X_{\text{dom}(i,r)} - \text{SIGMADOM}_{(i)} (p_{\text{roc}(i,r)} - p_{\text{dom}(i,r)})$$

$X_{\text{domimpf}(i,r)}$ is a demand for commodities from outside the local region

$P_{\text{roc}(i,r)}$ is a price of commodities from outside the local region

Domestic imports will increase (decrease) if domestic aggregate increases (decreases) and/or the price ratio changes in favour of domestic imports. The outcome depends also on the substitution possibilities.

¹¹ We now move to using only percentage change form or variables with lower-case letters. When convenient, we present at places also the level form.

In the fifth nest the producer chooses an optimal mix of domestic commodities and foreign imports. The logic of derivation is the same as for the nest four. The demand for domestic commodities aggregate and foreign imports are the following:

$$X_{\text{dom}(i,r)} = X_{\text{act}(i,r)} - \text{SIGMADOMROW}_{(i)} (p_{\text{dom}(i,r)} - p_{\text{act}(i,r)})$$

$$X_{\text{rowimp}(i,r)} = X_{\text{act}(i,r)} - \text{SIGMADOMROW}_{(i)} (p_{\text{rowimp}(i,r)} - p_{\text{act}(i,r)})$$

$X_{\text{act}(i,r)}$ is an Armington composite

$\text{SIGMADOMROW}_{(i)}$ is a CES substitution elasticity between domestic and foreign, imported commodities

$p_{\text{act}(i,r)}$ is a price of Armington composite

$p_{\text{rowimp}(i,r)}$ is a price of foreign imports in domestic currency

The interpretation is similar as in the case of local commodities and domestic imports.

The value of the Armington composite contains all production costs from all the five nests. This amount of output has been produced or supplied by using local, other domestic and foreign inputs.

There has to be demand that is equal to this supply in order for the economy to be in equilibrium. The CET (Constant Elasticity of Transformation) function is used to divide the Armington composite into three parts: commodities used locally, domestic exports and foreign exports.

The CET function is of the following form:

$$X_{\text{act}(i,r)} = (\bar{\delta}_{\text{xarm}(i,r)} X_{\text{arm}(i,r)}^{\rho(i)} + \bar{\delta}_{\text{domexp}(i,r)} X_{\text{domexp}(i,r)}^{\rho(i)} + \bar{\delta}_{\text{rowexp}(i,r)} X_{\text{rowexp}(i,r)}^{\rho(i)})^{1/\rho(i)}$$

$\bar{\delta}$ is a distribution parameter, $0 < \bar{\delta} < 1$

ρ is a transformation parameter, $\rho < 1, \neq 0$

$1 / (1 - \rho)$ is a transformation elasticity (ETRN)

The allocation for the three parts will be then as follows:

$$X_{\text{arm}(i,r)} = X_{\text{act}(i,r)} + \text{ETRN}_{(c)} (p_{\text{arm}(i,r)} - p_{\text{act}(i,r)})$$

$$X_{\text{domexp}(i,r)} = X_{\text{act}(i,r)} + \text{ETRN}_{(c)} (p_{\text{dom}(i,r)} - p_{\text{act}(i,r)})$$

$$X_{\text{rowexp}(i,r)} = X_{\text{act}(i,r)} + \text{ETRN}_{(c)} (p_{\text{rowexp}(i,r)} - p_{\text{act}(i,r)})$$

If the price ratio begins to favour one of the components, like foreign exports, then the allocation moves to the corresponding direction. The allocation follows also the supply of production or the Armington composite. The transformation possibilities have an effect too.

Primary factor markets

Labour markets are assumed to be regional, but the definition of the region can be selected from SAMs by using aggregation routines of the GEMPACK sub-program ViewHar. Labour is assumed to be homogenous, so there is only one profession or skill level.

Labour force

Labour force or the number of potential employed persons is defined in a usual way:

$$\text{EMPLOYED_I}_{(r)} = \sum_{(i,r)} \text{EMPLOYED}_{(i,r)}$$

$$\text{LABFORCE}_{(r)} = \text{EMPLOYED_I}_{(r)} / (1 - \text{UNEMPLOYRATE}_{(r)})$$

$\text{EMPLOYED_I}_{(r)}$ is number of employed persons

$\text{EMPLOYED}_{(i,r)}$ is a number of employed persons

$\text{LABFORCE}_{(r)}$ is a labour force

$\text{UNEMPLOYRATE}_{(r)}$ is unemployment rate

There is no connection between fixed labour supply and population. The change of the unemployment rate is defined:

$$\text{delUNEMPRAT}_{(r)} = 0.01 \left[\frac{\text{EMPLOYED_I}_{(r)}}{\text{LABFORCE}_{(r)}} \right] \left[\text{labrforce}_{(r)} - \text{employ}_{(r)} \right]$$

$\text{delUNEMPRAT}_{(r)}$ is change in unemployment rate

$\text{labrforce}_{(r)}$ is change in labour force

$\text{employ}_{(r)}$ is change of employed persons

Domestic net migration

Domestic regional net domestic migration is a linear function as follows:

$$\text{delNETMIG}_{(r)} = \alpha + \beta \text{delRELGDP}_{(r)} + \eta \text{delRELUNEMPRATE}_{(r)} + \text{dlambda}$$

α , β , η are coefficients from pooled cross-section time-series regression estimation

$\text{delNETMIG}_{(r)}$ is a change in persons moving to another region

$\text{delRELGDP}_{(r)}$ is a regional per capita GDP divided by national per capita GDP

$\text{delRELUNEMPRATE}_{(r)}$ is a regional unemployment rate minus national unemployment rate

dlambda is a slack variable chosen so that the sum of domestic regional migration is zero

It is assumed that the rise in the regional GDP differential correlates positively with the standard of living differential. The parameter β has a positive sign, so we assume a priori

that an increase in the standard of living in a certain region causes an increase in net migration (in-migration minus out-migration) towards that region. The unemployment differential, parameter η , is assumed to correlate negatively with net migration into the region. Therefore, the parameter for this term should have a negative sign.

When region's relative standard of living improves (diminishes) and/or its relative unemployment decreases (increases) more (less) people move there in a net terms. The outcome depends also on the parameter values.

Unemployment and wages

There is only one unbalanced market or quasi Walrasian element in the model. That is a labour market, which is assumed to be out of balance because of unemployment. This is nearly always a major problem in regional economies so needs to be modelled.

Relations for unemployment are as follows:

$$\text{UNEMPRATE}_{(r)} = \text{UNEMPLOYED}_{(r)} / \text{LABFORCE}_{(r)}$$

$$\text{unemprate}_{(r)} = \text{unemployed}_{(r)} - \text{labforce}_{(r)}$$

$$\text{delUNEMPLOY}_{(r)} = \text{UNEMPRATE}_{(r)} \text{ delLABFORCE}_{(r)} + \text{LABFORCE}_{(r)} \text{ delUNEMPRATE}_{(r)}$$

$$\text{delunemploy}_{(r)} = \text{UNEMPRATE}_{(r)} / 100 \text{ LABFORCE}_{(r)} \text{ labrforce}_{(r)} + \text{LABFORCE}_{(r)}$$

$$\text{delUNEMPRATE}_{(r)}$$

$\text{UNEMPRATE}_{(r)}$, $\text{unemprate}_{(r)}$ is unemployment rate

$\text{UNEMPLOYED}_{(r)}$, $\text{unemployed}_{(r)}$ is unemployed persons

$\text{delUNEMPLOY}_{(r)}$ is a change in unemployed

$\text{delLABFORCE}_{(r)}$ is a change in labour force

$\text{delUNEMPRATE}_{(r)}$ is a change in unemployment rate

$\text{delunemploy}_{(r)}$ is a change in unemployed persons

The change in unemployed persons depends on initial unemployment rate, and labour force, and their changes.

There are three theories for wage adjustment that the model can use. The first is to assume that labour unions have power to set wages. If wages are set too high, supply of labour is greater than demand, and there will be unemployment. This corresponds to a real wage equation:

$$p_{lab(i,r)} - p_{hou(r)} = flab_{(i,r)} + flab_{r(i)} + flab_{i(r)} + flab_{ir}$$

$p_{lab(i,r)}$ is a labour wage to worker

$p_{hou(r)}$ is consumer price index or inflation

$flab_{(i,r)}$, $flab_{r(i)}$, $flab_{i(r)}$, $flab_{ir}$ are real wage shifters for specific industry in a region, industry, region or nation

Shifter is a variable that is originally exogenous and not shocked, so has a zero value. In this case nominal wage follows inflation. This indicates sticky wages. The pair for the shifter is the corresponding nominal wage. When this is set exogenous and the shifter endogenous, the real wage equation will not be in effect.

The second theory assumes there is an adjusted Phillips curve for real wages:

$$p_{lab(i,r)} - p_{hou(r)} = \theta + \xi 100 (NAIRU_{(r)} - UNEMPRATE_{(r)}) - \pi 100 \text{delUNEMPRATE}_{(r)} + flab_{(i,r)} + flab_{r(i)} + flab_{i(r)} + flab_{ir}$$

θ is a parameter for secular technical progress (might be 2.5 %)

ξ , π are weight parameters

$NAIRU_{(r)}$ is a trend unemployment rate

Real wages and unemployment have a negative relation, but the outcome depends on three additional variables: secular technical progress, the difference between the trend and actual unemployment rates, and the change of the unemployment rates. Wages will decrease (increase) if initial unemployment rate is above (below) the trend unemployment rate. The change of the unemployment rate has a negative effect on wages. In general, real wages will fall (grow) when the labour market is loose (tight) indicating high (low) unemployment. Labour union power can be incorporated into the equation via real wage shifters. The outcome depends on the weight parameter values.

The third alternative is based on so called wage curve theory, Blanch-Flowe et al. (2006):

$$\text{delUNEMPRATE}_{(r)} = (P_{lab(i,r)} / P_{hou(r)})^{-\varphi}$$

$$\text{delunemprate}_{(r)} = -\varphi (p_{lab(i,r)} - p_{hou(r)}) + flab_{(i,r)} + flab_{r(i)} + flab_{i(r)} + flab_{ir}$$

Where: φ is an elasticity for the relationship

The assumption is that the unemployment rate and real wages are determined jointly. The expression in parentheses is a real wage. There is a negative relationship between unemployment rate and real wage. The equation states that when the labour markets become tight (loose), the demand for labour increases (decreases) and unemployment

decreases (increases), and real wages will increase (decrease). Labour union power can be incorporated into the equation via real wage shifters.

Capital and land markets

Aggregate capital consists of other physical capital and land in a CES technology (see Figure 3). Their markets are assumed to be both region and sector specific. This means that machines, equipment, tractors, lorries, buildings, wear houses etc. cannot be moved to another region or sold to another sector. Land is a production factor only in agriculture and forestry.

The demand for capital and land are exogenous in the short run, so their adjusting rents clear the markets. It is possible to set rents exogenous in the long run, so the demands are flexible and clear the markets. This is a traditional setting in short vs long run definition.

Taxation of primary factors and production

The model solves for taxation distorted equilibrium solutions. There are taxes on each of the primary factors, also production is taxed. All tax rates are average and added to the non-tax prices. A negative tax is defined as subsidy. All tax rates are exogenous in the model.

The relation between pre and post tax wages is the following:

$$\begin{aligned} \text{PLABPT}_{(i,r)} &= \text{PLAB}_{(i,r)} (1 + \text{LABTAXRATE}_{(i,r)}) \\ \text{PLABPT}_{(i,r)} \text{plabpt}_{(i,r)} &= \text{PLAB}_{(i,r)} (1 + \text{LABTAXRATE}_{(i,r)}) \text{plab}_{(i,r)} + 100 \text{PLAB}_{(i,r)} \\ &\text{delLABTAXRATE}_{(i,r)} \\ \text{PLAB}_{(i,r)} (1 + \text{LABTAXRATE}_{(i,r)}) \text{plabpt}_{(i,r)} &= \text{PLAB}_{(i,r)} (1 + \text{LABTAXRATE}_{(i,r)}) \text{plab}_{(i,r)} \\ &+ 100 \text{PLAB}_{(i,r)} \text{delLABTAXRATE}_{(i,r)} \\ (1 + \text{LABTAXRATE}_{(i,r)}) (\text{plabpt}_{(i,r)} - \text{plab}_{(i,r)}) &= 100 \text{delLABTAXRATE}_{(i,r)} \end{aligned}$$

$\text{PLABPT}_{(i,r)}$, $\text{plabpt}_{(i,r)}$ is a labour wage cost to firm

$\text{PLAB}_{(i,r)}$, $\text{plab}_{(i,r)}$ is a labour wage to worker

$\text{LABTAXRATE}_{(i,r)}$ is a labour tax rate

$\text{delLABTAXRATE}_{(i,r)}$ is a change of labour tax rate

The assumption is that the firm must pay more in wages than the worker receives, and the difference is the labour tax rate that the government has set.

The corresponding post tax capital and land rents for the firm are:

$$\begin{aligned} (1 + \text{CAPTAXRATE}_{(i,r)}) (\text{pcappt}_{(i,r)} - \text{pcap}_{(i,r)}) &= 100 \text{delCAPTAXRATE}_{(i,r)} \\ (1 + \text{LANDTAXRATE}_{(i,r)}) (\text{plandpt}_{(i,r)} - \text{pland}_{(i,r)}) &= 100 \text{delLANDTAXRATE}_{(i,r)} \end{aligned}$$

Here the owner's rent is lower than the user's rent.

The government receives labour tax revenue in the following way:

$$\begin{aligned} \text{VLABTAX}_{(i,r)} &= \text{VLAB}_{(i,r)} \text{LABTAXRATE}_{(i,r)} \\ \text{delLABTAX}_{(i,r)} &= (\text{VLABTAX}_{(i,r)} / 100) (\text{plab}_{(i,r)} + \text{xlab}_{(i,r)}) + \text{VLAB}_{(i,r)} \text{delLABTAXRATE}_{(i,r)} \end{aligned}$$

$\text{VLABTAX}_{(i,r)}$ is an initial labour tax revenue

$\text{VLAB}_{(i,r)}$ is an initial value of labour

$\text{delLABTAX}_{(i,r)}$ is a change of labour tax revenue

The first term on the right hand side is an initial value of the revenue after adjusting for the change in the tax base. The second term is an initial tax base adjusted for the change in the tax rate. The corresponding tax revenues for capital and land are:

$$\begin{aligned} \text{delCAPTAX}_{(i,r)} &= (\text{VCAPTAX}_{(i,r)} / 100) (\text{pcap}_{(i,r)} + \text{xcap}_{(i,r)}) + \text{VCAP}_{(i,r)} \text{delCAPTAXRATE}_{(i,r)} \\ \text{delLANDTAX}_{(i,r)} &= (\text{VLANDTAX}_{(i,r)} / 100) (\text{pland}_{(i,r)} + \text{xland}_{(i,r)}) \\ &+ \text{VLAB}_{(i,r)} \text{delLANDTAXRATE}_{(i,r)} \end{aligned}$$

The government earns more (less) when the use of factors increases (diminish), when their prices go up (down), or when the tax rates are increased (decreased).

Taxation and tax revenue of production follows the reasoning similar to primary factors. The relationship between pre and post tax output cost is:

$$(1 + \text{PRDTAXRATE}_{(i,r)}) (\text{ptot}_{(i,r)} - \text{pcost}_{(i,r)}) = 100 \text{delPRDTAXRATE}_{(i,r)}$$

$\text{PRDTAXRATE}_{(i,r)}$ is a production tax rate

$\text{ptot}_{(i,r)}$ is a production price of a sector including production tax

$\text{pcost}_{(i,r)}$ is an input cost of a sector excluding production tax

$\text{delPRDTAXRATE}_{(i,r)}$ is a change in production tax rate

Tax revenue from production is:

$$\text{delPRDTAX}_{(i,r)} = (\text{VPRDTAX}_{(i,r)} / 100) (\text{pcost}_{(i,r)} + \text{xtot}_{(i,r)}) + \text{VCOST}_{(i,r)} \text{delPRDTAXRATE}_{(i,r)}$$

$\text{delPRDTAX}_{(i,r)}$ is a change in production tax revenue

$\text{VPRDTAX}_{(i,r)}$ is an initial value of production taxes

$\text{xtot}_{(i,r)}$ is a sectoral output

$\text{VCOST}_{(i,r)}$ is a value of input cost of a sector excluding production tax

The first term on the right hand side is an initial value of the revenue after adjusting for the change in the tax base. The second term is initial tax base adjusted for the change in the

tax rate. The government earns more (less) when the production costs or volume increases (diminish), and/or when the production tax rate is increased (decreased).

Private consumption

There is one representative consumer or a household per region. It is assumed to be a utility maximizer. Utility refers to the total satisfaction received from consuming a basket of goods and services. The utility function describes the preferences. If the representative consumer behaves rationally she/he will choose the consumption bundle so that the utility is the greatest or maximized given the endowments/income available.

It should be noted that our model is of a “sector times sector” type. The consumers demand sectors’ products (like from food processing industry) directly, and there are no commodities or end products (like foodstuffs). The term commodity is used to specify goods and services.

Utility maximization problem of the consumer

The representative consumer maximises the Stone-Geary utility function:

$$\text{Max } U(X_{\text{tot}(i,r)1}, X_{\text{tot}(i,r)2}, \dots, X_{\text{tot}(i,r)n})$$

$$\text{subject to: } \sum_{(r)} P_{\text{lab}(i,r)} X_{\text{lab}(i,r)} + \sum_{(r)} P_{\text{cap}(i,r)} X_{\text{cap}(i,r)} + \sum_{(r)} P_{\text{land}(i,r)} X_{\text{land}(i,r)} = \text{WFACINC}_{(r)}$$

U is utility

$X_{\text{tot}(i,r)}$ is a demand for sectoral output

$\text{WFACINC}_{(r)}$ is a factor income of the consumer

Utility is maximized subject to the preferences, and the budget constraint, which demands that the value of total consumption must be equal to consumer’s factor income. The optimal solution is reached when:

$$\text{MRS}(X_{\text{tot}(i,r)i}^*, X_{\text{tot}(i,r)j}^*) = P_{\text{tot}(i,r)i} / P_{\text{tot}(i,r)j}, \text{ for all } i \neq j$$

MRS is a marginal rate of substitution

* indicates the optimal demand

On the left hand side we have a rate at which the consumer is willing to exchange commodities. On the right is a rate which the commodity market is willing to exchange commodities. Utility is maximized if these rates are equal for all commodities.

Factor income

The representative consumer owns all primary factor endowments, and collects all factor earnings from wages, capital and land rents. The factor income becomes:

$$VFACINC_{(r)} wfacinc_{(r)} = \sum_i VLABINC_{(ir)} wlabinc_{(ir)} + \sum_i VCAPINC_{(ir)} wcapinc_{(ir)} + \sum_i VLANDINC_{(ir)} wlandinc_{(ir)}$$

$$VLABINC_{(ir)} = VLAB_{(ir)} (xlab_{(i,r)} + plab_{(i,r)})$$

$$VCAPINC_{(ir)} = VCAP_{(ir)} (xcap_{(i,r)} + pcap_{(i,r)})$$

$$VLANDINC_{(ir)} = VLAND_{(ir)} (xland_{(i,r)} + pland_{(i,r)})$$

$VFACINC_{(r)}$ is an initial value of factor income

$wfacinc_{(r)}$ is change of factor income

$VLABINC_{(ir)}$, $wlabinc_{(ir)}$, $VCAPINC_{(ir)}$, $wcapinc_{(ir)}$, $VLANDINC_{(ir)}$, $wlandinc_{(ir)}$ are respectively labour, capital and land income

$VLAB_{(ir)}$, $VCAP_{(ir)}$, $VLAND_{(ir)}$ are initial values of labour, capital and land respectively

Taxation is not yet included, so the equation describes net income which consists of labour, capital and land income. Factor income will grow (diminish) if the demand for factors and/or their price increases (decreases).

Income taxes, transfers and disposable income

The consumer pays direct income tax, as well as value-added tax and other indirect consumption taxes to the government. There are also income transfers, such as pensions, unemployment and other benefits etc., from the government to the consumer. All taxes and transfers are calculated using average tax rates which are exogenous. We are calculating them for region, because there is only one representative consumer per region. Labour and land income usually stay in the region, but capital income tends to flow outside the region. For simplicity, we assume that all capital income goes into national pot and is distributed to regional consumers in proportion to regional GDPs.

The capital income is of the form:

$$SHRGDP_{(r)} = WGDPEXP_{(r)} / WGDPEXP_{ir}$$

$$shrgdp_{(r)} = wgdpexp_{(r)} - wgdpexp_{ir}$$

$$wcapinc_{(r)} = shrgdp_{(r)} wcapinc_{ir}$$

$SHRGDP_{(r)}$, $shrgdp$ is a regional share of GDP

$WGDPEXP_{(r)}$, $wgdpexp$ is a regional nominal expenditure side GDP

$WGDPEXP_{ir}$, $wgdpexp_{ir}$ is a national nominal expenditure side GDP

$wcapinc_{(r)}$ is a capital income to regional consumer

$wcapinc_{ir}$ is a national capital income

The regional share of GDP is dependent on the ratio of regional and national GDP. It can decrease, stay the same, or increase depending on what happens to the nominal regional GDP. Regional capital income is this share times the national capital income from the common pot.

Direct transfers to the consumer are:

$$\begin{aligned} WGOVHOU_{(r)} &= WGDPEXP_{(r)} FGOVHOU_{(r)} \\ wgovhou_{(r)} &= wgdpexp_{(r)} + fgovhou_{(r)} \end{aligned}$$

$WGOVHOU_{(r)}$, $wgovhou_{(r)}$ are transfers from government to the regional consumer
 $FGOVHOU_{(r)}$, $fgovhou_{(r)}$ are shifters for transfers from government to the regional consumer

The assumption is that the transfers from the government follow GDP. The shifter can be used to exogenize the level of the transfers.

Income taxes from the consumer to the government are:

$$\begin{aligned} VHOUGOV_{(r)} &= THOUGOV_{(r)} VHOUSINC_{(r)} \\ whougov_{(r)} &= thougov_{(r)} + whousinc_{(r)} \\ VHOUSINC_{(r)} whousinc_{(r)} &= VLABINC_{(r)} wlabinc_{(r)} + CAPTINC_{(r)} wcapinc_{(r)} \\ &+ VLANDINC_{(r)} wlandinc_{(r)} + VGOVHOU_{(r)} wgovhou_{(r)} \end{aligned}$$

$VHOUGOV_{(r)}$, $whougov_{(r)}$ = income tax from the consumer to the government
 $THOUGOV_{(r)}$, $thougov_{(r)}$ = income tax rate
 $VHOUSINC_{(r)}$, $whousinc_{(r)}$ = income of the consumer
 $VGOVHOU_{(r)}$, $wgovhou_{(r)}$ = transfers from government to the consumer

Income tax is the tax rate times the tax base, including the factor income and transfers. The latter ones are subject to income taxation, and considered as normal income.

Finally:

$$\begin{aligned} VHOUSNETINC_{(r)} whousnetinc_{(r)} &= VHOUSINC_{(r)} whousinc_{(r)} - VHOUGOV_{(r)} whousgov_{(r)} \\ VHOUSNETINC_{(r)}, whousnetinc_{(r)} & \text{ is disposable income of the consumer} \end{aligned}$$

Indirect consumption taxes

The consumer uses local and imported commodities from the other domestic regions and from foreign countries. There is an indirect consumption tax for the two first ones, but foreign imports are exempted from taxation. Local consumption is taxed as follows:

$$(1 + \text{HOUTAXRATE}_{(r)}) (\text{parmhpt}_{(c,r)} - \text{parm}_{(c,r)}) = 100 \text{delHOUTAXRATE}_{(r)}$$

$\text{HOUTAXRATE}_{(r)}$, $\text{delHOUTAXRATE}_{(r)}$ are tax rates of local and domestically imported commodities respectively

$\text{parmhpt}_{(c,r)}$ is a price of local commodities for the consumer including tax

$\text{parm}_{(c,r)}$ is a price of local commodities excluding tax

The consumer pays prices with add-valorem indirect taxes, according to an average tax rate determined by the government. Domestic imports are taxed similarly:

$$(1 + \text{HOUTAXRATE}_{(r)}) (\text{prodhpt}_{(c,r)} - \text{prod}_{(c,r)}) = 100 \text{delHOUTAXRATE}_{(r)}$$

$\text{prodhpt}_{(c,r)}$ is a price of domestically imported commodities for the consumer including tax

$\text{prod}_{(c,r)}$ is a price of domestically imported commodities for the consumer excluding tax

The government earns tax revenue from consumption:

$$\begin{aligned} \text{delHOUTAX}_{(r)} = & \text{HOUTAXRATE}_{(r)} / 100 \sum_{(r)} \text{VHOU}_{(c,r)} (\text{parm}_{(c,r)} + \text{xhou}_{(c,r)}) \\ & + \text{HOUTAXRATE}_{(r)} / 100 \sum \text{VHOUROD} (\text{prod}_{(c,r)} + \text{xhrod}_{(c,r)}) \\ & + \sum_{(r)} \text{VHOU}_{(r)} \text{delHOUTAXRATE}_{(r)} + \sum_{(r)} \text{VHOUROD}_{(r)} \text{delHOUTAXRATE}_{(r)} \end{aligned}$$

$\text{delHOUTAX}_{(r)}$ is a tax revenue from local and domestically imported commodities

$\text{VHOU}_{(c,r)}$ is an initial value of local consumption

$\text{xhou}_{(c,r)}$ is a real demand for local commodities

$\text{VHOUROD}_{(r)}$ is an initial value of domestically imported consumption

The tax revenue will increase (decrease) if the prices and/or demand for local and/or domestically imported commodities grow (diminish). Also, the change of the tax rate influences the development of the tax revenue.

Stone-Geary specification

The consumer uses local and imported commodities from the other domestic regions and from abroad. The preference for these commodities is expressed with a Stone-Geary (or Klein-Rubin) utility function:

$$U = [(X_{tot(c,r)1} - \xi_1)^{\omega_1} (X_{tot(c,r)2} - \xi_2)^{\omega_2}, \dots, (X_{tot(c,r)n} - \xi_n)^{\omega_n}]$$

ξ is a subsistence demand of sectoral output

ω is an expenditure share of sectoral output

The feature of this functional form is that the subsistence and supernumerary or luxury commodities can be defined. It also gives rise to a LES (Linear Expenditure System), which contain no substitution effects. Only income effects appear in the system, so only the budget shares of commodities, expenditure elasticities, and a Frisch parameter are needed.

The demand functions are of the form:

$$X_{tot(c,r)1} = \xi_1 + (\omega_1 / P_{tot(c,r)1}) (WHOU_{(r)} - \xi_1 P_{tot(c,r)1} - \xi_2 P_{tot(c,r)2}, \dots, - \xi_n P_{tot(c,r)n})$$

Where: $WHOU_{(r)}$ is a total expenditure

Subsistence means that a certain minimal level of some good has to be consumed, irrespective of its price or the consumer's income. The consumer gets utility from a commodity only to the extent that its consumption exceeds the subsistence level.

The demand for subsistence and supernumerary commodities, after introducing the number of households and taste changes are:

$$x_{sub(c,r)} = n_{hou(r)} + a_{sub(c,r)}$$

$$a_{sub(c,r)} = a_{hou(c,r)_s} - \sum_{x_{tot(c,r)1}, \dots, n} (BUDGSHR a_{hou_s})$$

$x_{sub(c,r)}$ is a subsistence demand of sectoral output

$n_{hou(r)}$ is a number of households

$a_{sub(c,r)}$ is a taste change shifter for subsistence demand

a_{hou_s} is a taste change shifter between domestic and imported commodities

BUDGSHR is an average budget share of the consumer

We are assuming that the subsistence quantity of each commodity is constant, so independent of consumer prices. This means that the subsistence demands follow the number of households, and there might be some taste changes between domestic and imported commodities. Subsistence demand increases (decreases) if the number of

households grows (diminishes) or the taste changes in favour (unfavour) of domestic commodities.

$$x_{lux(c,r)} = w_{lux(r)} - p_{hou(r)} + a_{lux(c,r)}$$

$$V_{HOUTOT(r)} p_{hou(r)} = \sum_{1, \dots, n} V_{HOUP T(c,r)} p_{armhpt(c,r)} + \sum_{1, \dots, n} V_{HRODPT(c,r)} p_{rodhpt(c,r)} + V_{FORIMP(c,r)} p_{forimp(c,r)}$$

$$a_{lux(c,r)} = a_{sub(c,r)} - \sum_{x_{tot1, \dots, n}} (SLUX_{(c,r)} a_{sub(c,r)})$$

$$SLUX_{(c,r)} = -EPS_{(c,r)} BUDGSHR_{(c,r)}$$

$x_{lux(c,r)}$ is a supernumerary demand of sectoral output

$p_{hou(r)}$ is a consumer price index, inflation

$w_{lux(r)}$ is a total nominal supernumerary expenditure of the consumer

$a_{lux(c,r)}$ is a taste change shifter for supernumerary demand

$V_{HOUTOT(r)}$ is an initial total spending of the consumer

$V_{HOUP T(c,r)}$, p_{armhpt} is an initial value and price of local commodities including tax

$V_{HRODPT(c,r)}$, p_{rodhpt} is an initial value and price of domestically imported commodities including tax

$V_{FORIMP(c,r)}$, p_{forimp} is an initial value and price of foreign imported commodities in domestic currency

$SLUX_{(c,r)}$ is a marginal budget share of the consumer

$EPS_{(c,r)}$ is an expenditure elasticity of the consumer

$BUDGSHR$ is the average budget share

The consumer trades with prices that are weighted average of local, domestically and foreign imported commodities. Supernumerary demands follow the corresponding real consumer expenditure, and there might be some taste changes between domestic and imported commodities. The expenditure elasticity of each commodity affects the allocation of taste changes.

$$x_{hou(c,r)} = BLUX_{(c,r)} x_{lux(c,r)} + (1 - BLUX_{(c,r)}) x_{sub(c,r)}$$

$$BLUX_{(c,r)} = -EPS_{(c,r)} / FRISCH_{(r)}$$

$$x_{houtot(r)} = \sum_{x_{tot1, \dots, n}} (BUDGSHR_{(c,r)} x_{hou(c,r)})$$

$x_{hou(c,r)}$ is a demand for commodities from the consumer

$BLUX_{(c,r)}$ is a ratio between supernumerary expenditure and total expenditure

$FRISCH_{(c)}$ is a LES parameter which equals - total expenditure divided by supernumerary expenditure

$x_{houtot(r)}$ is a real spending of the consumer

The consumer demands both the subsistence and supernumerary commodities. The allocation is dependent of the commodity specific expenditure elasticities of the consumer and the Frisch parameter. Finally, the real spending of the consumer is a budget share weighted sum of the demands for the commodities.

Public consumption

The public sector does not produce its services, but buys them from the firms or the private sector. The government demands only local commodities, and they are not taxed.

Public budget

The government collects both indirect and direct taxes. The former are additions to prices, and the latter are income taxes that reduce the disposable income of the consumer. There are also transfers from the government to the regional consumers. The change in the public budget, delGOVBUDJ is:

$$\begin{aligned} \text{delGOVBUDJ}_{(r)} &= \sum_{(i)} \text{delLABTAX}_{(i,r)} + \sum_{(i)} \text{delCAPTAX}_{(i,r)} + \sum_{(i)} \text{delLANDTAX}_{(i,r)} \\ &+ \sum_{(i)} \text{delPRDTAX}_{(i,r)} + \text{delHOUTAX}_{(r)} + \text{delINVTAX}_{(r)} - \text{VGOVHOU}_{(r)} \text{wgovhou}_{(r)} \\ &+ \text{VHOUGOV}_{(r)} \text{whougov}_{(r)} \end{aligned}$$

All tax forms are expressed in monetary value terms. The revenues from the use of primary factors, output, consumption, investments, and the income taxes increase the public budget. Transfers are expenditures for the government.

Public spending

The price index of the government is:

$$\text{VGOVTOT}_{(g,r)} \text{pgov}_{(g,r)} = \sum_{1, \dots, n} \text{VGOVTOT}_{(g,r)} \text{parm}_{(c,r)}$$

$\text{VGOVTOT}_{(g,r)}$ is an initial value of government spending

$\text{pgov}_{(g,r)}$ is a government price index

$\text{parm}_{(d,r)}$ is a price of local consumption

The government buys with prices that are a weighted sum from the local consumption prices. Public sector spending is defined as:

$$\text{VGOVTOT}_{(g,r)} \text{xgovtot}_{(g,r)} = \sum_{1, \dots, n} \text{VGOVTOT}_{(g,r)} \text{xgov}_{(c,g,r)}$$

Where: $\text{xgovtot}_{(g,r)}$ is a real government spending on commodities

$\text{xgov}_{(c,g,r)}$ is a government demand for local commodities

This leads to:

$$\text{wgovtot}_{(g,r)} = \text{pgov}_{(g,r)} + \text{xgovtot}_{(g,r)}$$

$\text{wgovtot}_{(g,r)}$ is a value of government spending on commodities

The value of government spending will increase (decrease) if the price level and/or demand grows (diminishes).

Policy tools

All tax rates are average and added to the non-tax prices. A negative tax is a subsidy. The government has seven exogenous policy tools:

$\text{delLABTAXRATE}_{(i,r)}$ - change of labour tax rate

$\text{delCAPTAXRATE}_{(i,r)}$ - change of capital tax rate

$\text{delLANDTAXRATE}_{(i,r)}$ - change of land tax rate

$\text{delPRDTAXRATE}_{(i,r)}$ - change of output tax rate

$\text{del INVTAXRATE}_{(r)}$ - change of investment tax rate

$\text{delHOUTAXRATE}_{(r)}$ - change of consumption tax rate

$\text{thousgov}_{(r)}$ - a national income tax rate

The government can adjust the tax rates and hence to change the relative competitiveness of factors and output among the sectors. The four first are in effect for each sector and region. The investment and consumption tax rates are only regional. This means that the change in the tax rate will be the same for all commodities used and consumed. The income tax rate is also per region.

Investment

Only aggregate or gross investments are modelled. There is no separation between private and public, and the changes of the inventories are included in the concept.

Investments are exogenous, so they can be shocked. In this setting, there is only a weak link between investments and the demand for capital. If investments are increased (decreased), the consumer must consume less (more), so save more (less) in order to balance the budget.

This might have effects on production, so it is possible that the demand for capital can change. In a comparative-static model there cannot be any investment-capital stock accumulation mechanism like in the dynamic inter-temporal models. The capital stock or the supply of capital does not change, so the capital markets are cleared by adjusting rents or price of capital.

Domestic and foreign trade

All regions trade at the domestic and foreign markets. Domestic and foreign imports are expenditure, and the corresponding exports are income for the region or the representative consumer.

Domestic trade¹²

The formulation is simple, and was based on scarce availability of data. The trade matrix for a region is diagonal, so we know only the aggregate values, not traded values from different sectors. The consumer, producers, and the investors demand domestic imports.

$$VROCIMP_{(ir)} xrocimp_{(ir)} = VHOU_{(ir)} xhroc_{(ir)} + VINT_{(ir)} xrocimpf_{(ir)} + VINV_{(ir)} xiroc_{(ir)}$$

$VROCIMP_{(ir)}$, $xrocimp_{(ir)}$ is initial value, and total demand for rest of country (ROC) commodities

$VHOU_{(ir)}$, $xhroc_{(ir)}$ is initial value, and consumption demand for ROC commodities

$VINT_{(ir)}$, $xrocimpf_{(ir)}$ = initial value, and demand from production for ROC commodities

$VINV_{(ir)}$, $xiroc_{(ir)}$ = initial value, and demand from investment for ROC commodities

The total or aggregate demand for ROC commodities is a weighted sum of the demand from the three actors. Only the demand from production is based on output and relative prices, as we saw in the description of the nested Armington structure. The three demands are endogenous, and the aggregate demand increases (decreases) if one or more of the three demanders increase (decrease) their usage.

Domestic export depends on output and relative prices and was explained in the CET part of the Armington structure.

Foreign trade

The demand for foreign imports is dependent on output and relative prices, and is described in the Armington structure. Foreign export depends on output and relative prices and was explained in the CET part of the Armington structure.

Foreign trade can be in deficit, balance or in surplus. This comes from the assumption of small open economy, which is assumed to be able to trade with exogenous world prices without any restriction from world markets. Only the price ratios toward local and ROC commodities and their substitution possibilities affect which share of total output is allocated to foreign exports.

Equilibrium conditions

Up till now, we have presented definition and behaviour equations. In an equilibrium model there must be also conditions for clearance of all markets. The commodity, factor, and markets for domestic trade must be in balance. The restriction is looser for foreign trade. The exception is the labour markets, where we allow for unemployment.

¹² Domestic trade is in balance by definition, we return to the condition in the next chapter.

Commodity markets

The requirement for equilibrium allocation is that for all commodities, supply equals demand. The tax inclusive prices indicate both the amount received by sellers and the amount paid by buyers per unit of the associated commodity. For each class of produced commodities, there are three associated markets, one for commodities produced for the local market, another for domestic and foreign export and a third for import markets. Commodities produced for the local market enter intermediate, consumer, government and investment demand, and must satisfy the following condition:

$$Xarm_{(ir)} = Xint_{(ir)} + Xhou_{(r)} + Xgov_{(g)} + Xinv_{(ir)}$$

One can see supply on the left hand side, and demand on the right hand side. The condition says that supply and demand for all commodities must be equal in the equilibrium. The corresponding prices will adjust until the condition is met.

Factor markets

In order for the capital markets to clear, we must first set a restriction stating: investment and saving (income minus consumption) have to match, so:

$$Winv_{(r)} = Whousnetinc_{(r)} - Whou_{(r)}$$

$Whousnetinc_{(r)}$ = disposable income of the consumer

$Whou_{(r)}$ = spending by the consumer

This means that investments are financed from savings. This also means that the consumer lives on a balanced budget, so has paid her/his taxes and uses the net income in consumption and savings.

The equilibrium condition for the capital and land markets is:

$$Xcap_ir = \sum_{(ir)} Xcap_{(ir)}$$

$$Xland_ir = \sum_{(ir)} Xland_{(ir)}$$

$Xcap_ir$, $Xland_ir$ are total capital and land endowments

The producers cannot use more than the total amount of factor endowments.

The labour markets are out of balance due to unemployment. The demand-supply condition is:

$$Xlab_ir = \sum_{(i,r)} Xlab_{(ir)} (1 - UNEMPRATE_{(r)})$$

$Xlab_ir$ is total labour endowment

$UNEMPRATE_{(r)}$ is unemployment rate

Supply of labour or endowment is restricted because only the employed from the labour force are available for demand.

Domestic and foreign trade

The market for domestic imports is analogous to the market for domestic output, and the supply-demand balance (also by definition) is written:

$$\sum_{(r)} Xrodimp_{(i)} = \sum_{(r)} Xrodexp_{(i)}$$

$$Xrodimp_{(ir)} = Xhrodimp_{(ir)} + Xrodimpf_{(ir)} + Xirodimp_{(ir)}$$

$$Xrodexp_{(ir)} = Xrodexpf_{(ir)} + Xhrodexp_{(ir)}$$

$Xhrodimp_{(i)}$ is demand and supply from consumption for imports and exports

$Xrodimpf_{(ir)}$ is demand and supply from production for imports and exports

$Xirodimp_{(ir)}$ is demand from investors for domestic imports

In foreign trade there is a more flexible situation:

$$\sum_{(r)} Xrowimp_{(i)} \Leftrightarrow \sum_{(r)} Xrowexp_{(i)}$$

$Xrowimp_{(ir)}$ is demand for foreign imports

$Xrowexp_{(ir)}$ is supply of foreign exports

Macros

Our model is built to perform from bottom-up. The regions are the bottoms, and the model is solved for all regions simultaneously. This model type emphasises regional results, but do not rule out national outcomes because they can be aggregated from the regional results. Let's take some examples:

$$\begin{aligned} VGDPEXP_{(r)} xgdpexp_{(r)} &= VHOUTOT_{(r)} xhoutot_{(r)} + VGOVTOT_{(r)} xgovtot_{(r)} \\ &+ VINVTOT_{(r)} xinvtot_{(r)} + VROCEXPTOT_{(r)} xrocexptot_{(r)} + VROWEXPTOT_{(r)} xrowexptot_{(r)} \\ &- VROCIMPTOT_{(r)} xrocimptot_{(r)} - VROWIMPTOT_{(r)} xrowimptot_{(r)} \end{aligned}$$

$VGDPEXP_{(r)}$, $xgdpexp_{(r)}$ is initial value and change of expenditure side GDP

$VHOUTOT_{(r)}$, $xhoutot_{(r)}$ is initial total value and change of private consumption

$VGOVTOT_{(r)}$, $xgovtot_{(r)}$ is initial total value and change of public consumption

$VINVTOT_{(r)}$, $xinvtot_{(r)}$ is initial total value and change of investments
 $VROCEXPTOT_{(r)}$, $xrocexptot_{(r)}$ is initial total value and change of domestic exports
 $VROWEXPTOT_{(r)}$, $xrowexptot_{(r)}$ is initial total value and change of foreign exports
 $VROCIMPTOT_{(r)}$, $xrocimptot_{(r)}$ is initial total value and change of domestic imports
 $VROWIMPTOT_{(r)}$, $xrowimptot_{(r)}$ is initial total value and change of foreign imports

Regional GDP is a weighted sum of its components. National GDP, $xgdpexp_r$ is defined:

$$\sum_r VGDPEXP_{(r)} (xgdpexp_{(r)} - xgdpexp_r) = 0$$

The same holds for national GDP's components. In the same way for instance:

$$\begin{aligned} \sum_{(ir)} VLABPT_{(r)} (xlab_{(r)} - xlab_ir) &= 0 \\ \sum_{(ir)} VCAPPT_{(r)} (xcap_{(r)} - xcap_ir) &= 0 \\ \sum_{(ir)} VLANDPT_{(r)} (xland_{(r)} - xland_ir) &= 0 \end{aligned}$$

for national total labour, capital, and land use. The national sector output, $xtot_r$ is:

$$\sum_{(r)} VPRIM_{(r)} (xtot_{(r)} - xtot_r) = 0$$

The same procedure can be used for any macro variable that is of interest.

Conclusions

This paper discussed a methodology of the model CGERegEU. After presenting an overview on the regional CGE modelling in general, we concluded that our approach fits well in the mainstream of economic CGE analysis. Our model is well rooted in the tradition of neoclassical theory, and it was also tested in practice in many real life applications in Finland and Poland. Hence, it is a good starting point for our further work in constructing the layer of single-country CGE models for CAPRI. We also showed how the two modelling traditions: level and linearization schools can be substitutes. They are easily transferable by using simple transformation rules. The structure was discussed in details by presenting the equations.

The model has four new features comparing to the version proposed in the application. They are: land factor, different real wage theories, the Stone-Geary utility function (instead of Cobb-Douglas) and public sector accounting. Thanks to these additions we have now three primary production factors: labour, capital, and land. Besides, now we can experiment with different wage theories: sticky wages, adjusted Phillips curve or Wage curve based real wages. The Cobb-Douglas utility function has been replaced with the Stone-Geary leading to Linear Expenditure System, which is more sophisticated. Public sector accounting shows the tax revenues collected, and transfers given.

The next step will be to construct the database which will suit all the requirements of the CGERegEU construct. It means building regional SAMs at NUTS 2 regions for EU27+.

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Annex 1: Examples of linearisation rules

Percentage Change Examples

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Levels form	100*ordinary change	Percentage change
X	Xx	$x = 100\Delta X/X$
$3X$	$3Xx$	x
XY	$XY(x+y)$	$x+y$
X/Y	$(X/Y)(x-y)$	$x-y$
X^α	$X^\alpha(\alpha x)$	αx
$X+Y$	$Xx+Yy$	$(Xx+Yy)/(X+Y)$ $= S_x x + S_y y$ where $S_x = X/(X+Y)$, $S_y = 1-S_x$
<p>In the examples below we assume a vector of quantities X_i, total X. a vector of prices P_i, average P. a vector of values V_i, total V, such that $V_i = P_i X_i$, (thus $v_i = p_i + x_i$). that $V = PX$, (thus $v = p + x$).</p>		
Adding up quantities (rare)		
$X = \sum X_i$	$Xx = \sum X_i x_i$	$x = \sum S_i x_i$ where $S_i = X_i/X$
Adding up quantities with a common price P (frequent)		
$PX = P\sum X_i$	$PXx = \sum P X_i x_i$ or $Vx = \sum V_i x_i$ or $\sum V_i(x-x_i) = 0$	$x = \sum S_i x_i$ where $S_i = V_i/V$
Adding up values		
$V = \sum V_i$	$Vv = \sum V_i v_i$ or $V(p+x) = \sum V_i(p_i+x_i)$	$v = \sum S_i(p_i+x_i)$ where $S_i = V_i/V$
Decomposing value changes into indices of price and quantity change		
$V = \sum V_i$	$V(p+x) = \sum V_i(p_i+x_i)$	from $v = \sum S_i(p_i+x_i)$ price part: $p = \sum S_i p_i$ quantity part: $x = \sum S_i x_i$ so $v = p + x$
CES (no tech change)		
$X = CES(X_i)$		$x_i = x - \sigma(p_i-p)$ where $p = \sum S_i p_i$ implying $x = \sum S_i x_i$
CES (with tech change, $a_i < 0$ for progress)		
$X = CES(X_i/A_i)$		$x_i - a_i = x - \sigma(p_i + a_i - p)$ where $p = \sum S_i(p_i + a_i)$ implying $x = \sum S_i(x_i - a_i)$
Ordinary change in a ratio		
$R = X/Y$	$Rr = 100\Delta R = (X/Y)(x-y)$ $100\Delta R = R(x-y)$	$100(\Delta R)/R = r = x - y$