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**Calibration of feed requirements and
price determination of feed in CAPRI**

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Abstract

The introduction of this paper describes general problems in using a primal solution of deriving feed cost, based on requirement constraints and feeding activities. Based on the discussion, the first part paper describes a calibration process for requirements based on Maximum Entropy Econometrics. The second part discusses how trading prices for fodder can be derived in the context of a regionalised programming model. Positive Mathematical Programming is subsequently used to "fine tune" the model which is described in part 3. A summary with a description on how the suggested solutions integrate within the overall model concept concludes the paper.

1 Introduction

1.1 Warming up: Style questions in feed gymnastics

Feed use determines to a large extent costs in animal production as well as total use of certain crops. Hence, from the view point of policy modelling, a plausible description of the relations between animal production, feed use and market prices for feeds is necessary.

The issue has been addressed by a broad range of methodological solutions in sector models. Some important solutions are characterised as follows:

- (1) *Products used for feed are modelled as net-puts*, only, depending on crop and animal prices and further factors, e.g. technological progress and availability of primary factors. Hence, the net-put approach models feed use implicitly. Typically, the relevant parameters are estimated based on duality in the context of a profit function where the production possibility set is hidden. Plausibility checks of these hidden technological relations are hard to do and still harder to incorporate in the estimation approach.
- (2) *Feed use is modelled explicitly, but the underlying technology is hidden*, e.g. when cost functions for the feed compound industry are estimated as in BRITZ & SIEBER (1998). Modelling feed use as a function of prices is quite common, for example in multi-commodity models such as WATSIM of the IAP or the World Food Model of the FAO. The technological relations are hidden as in (1), but somewhat easier to check by comparing the change in animal production provoked by a price change with its effects on feed use.

The next solutions refer to programming models:

- (3) *Feed use is modelled as fix cost per unit of animal production activities*. Animal production will not be affected by changes in crop production and vice versa.
- (4) *Feed use is modelled via feeding activities*. In that case, the model typically simultaneously determines the optimal levels of animal and crop production and the amount of each output fed to the animals. These solutions differ by:
 - a) The *aggregation level of the feedingstuff*. Some models, for example the German sector model RAUMIS, have feeding activities for each output to each animal activity, excluding technological impossible combinations such as feeding straw to piglets. The solution typically leads to a rather high number of feeding activities.

Others, as the SPEL MFSS, use aggregates of raw products in the feeding activities. Last but not least, single raw products can be mixed to different predefined menus whose mix is then the endogenous variable, as in TASM.

- b) The *definition of the constraints*. In RAUMIS and SPEL-MFSS (WEBER 1995, pp. 39), requirements such as energy or protein are modelled explicitly. If mixes for certain animals are defined beforehand, explicit requirement constraints in the model may be left out if each individual mix guarantees already that requirements are covered. The higher the number of feed use activities per animal activity, and the lower the number of requirement constraints, the higher the chance of strongly overspecialised solutions. In order to ensure a "plausible" mix of the feeding activities, bounds on the feeding activities are sometimes used, as in SPEL-MFSS or in earlier versions of RAUMIS. The substitution possibilities may be influenced by using PMP on feeding activities, as in the new version of RAUMIS (CYPRIIS 1999).

Solution (3) and (4) may be iteratively coupled to modules which determine the costs for solution (3) or prices used in solution (4).

1.2 Specific exercises for CAPRI

Such an iterative coupling is the case in CAPRI which addresses the question of modelling the relations between animal and crop production, price and markets as follows:

- Feed use of non-tradable fodder such as graze is modelled by individual feeding activities in the regional supply models whereas feeding activities of tradable products such as wheat or soy beans are aggregated to five categories (cereals, rich protein, rich energy, milk based and others).
- Requirement constraints are introduced to ensure technological plausible substitution between feeds.
- Calibration to observed national feed use is guaranteed by PMP calibration terms.
- Tradable feedingstuffs can be sold and bought in unlimited quantities. No difference is made between tradable feedingstuffs produced in the region and such bought (net trade approach).
- Hay and straw are assumed to be tradable only inside a Member State according to the definition of the Economic Accounts of Agriculture. Regional trading prices are functions of the quantities sold or bought to account for transport and transaction costs.
- The regional models are solved independently from each other with prices for outputs and inputs - including feedingstuff - fixed.
- Regional net trade from the regional supply model is aggregated per Member State and used as exogenous variables in the market module including the quantities of the five feed aggregates. Market clearance is achieved by behavioural functions who describe final demand of all member states and rest-of-the-world and supply of rest-of-the world as a function of prices and shift factors such as income and population. Substitution inside of the given quantities of the five feed aggregates is modelled by elasticities (for details of the market module, see BRITZ 1998).
- Market module and regional supply models are iteratively linked where prices from the market module are used in the supply module generating quantities that in turn are used in the market module. For the five feed aggregates, the market module determines - with

each iteration a new price and new nutrient contents for the feed aggregates (for details see: HECKELEI, BRITZ & LOEHE 1998).

The subsequent sections of this paper are mostly devoted to the question how the regional models embedded in the specific CAPRI layout described above can be specified in a way that

§ *calibration* of observed feeding quantities is achieved, and a

§ *plausible simulation behaviour* for feed use is obtained.

2 Statistical reality meets linear programming

2.1 First round: Let's prove that reality is sub-optimal ...

An important feature of the CAPRI model is a rather explicit, primal modelling of feeding activities in the supply part as a cost minimising problem. Substitution possibilities in feeding are modelled by requirement constraints for each animal category which can be satisfied by an appropriate feed mix. The approach is common in programming models because:

1. It is known that the feed compound industry, extension specialists and farmers use programming models to minimise their feed costs. Hence, there is hope that a simplified and aggregated version of such a model will work in a sectoral context as well.
2. Information on nutrient requirements of animals are published in the literature as are nutrient contents of feedingstuffs, so that the constraint matrix can be specified. Prices for outputs including feedingstuffs are anyway needed in the context of a sector model.

The drawback of the approach is a long standing experience of modellers with overspecialised solutions from such feed cost minimising models and a long lists of pragmatic tricks to get rid of them. Some typical problems of aggregate models apply here as well: some data such as availability and quality of fodder are much harder to get on the sectoral than at single farm level. Published nutrient requirements of animals often relate to controlled experiments, and not to the actual practise. Specific constraints included in farm models cannot be incorporated, and the specification of models used by the feed compound industry are not published.

When running a simulation with a typical sector model, the determination of the crop rotation, animal herd sizes and feeding practise is a simultaneous problem.¹ We will resist here discussing how activities levels in crop and animal production are calibrated in CAPRI (see HECKELEI 1997 and HECKELEI, BRITZ 1999) and assume for simplicity that herd sizes (LEVL) of animal categories (a) are given and all feedingstuffs (f) are bought at known prices (PRICE).

In order to shed light on the relation between overspecialisation and calibration, we will start with the assumption that physical requirements (REQU) for energy, protein etc. are known as well as the nutrient contents (NUTR) of the feedingstuff. This information serves to define the constraints ("c") of our feed cost minimising problem which is formulated for one of the regions ("r") as follows:

$$\begin{aligned}
 & \min \sum_{a,f} \text{FEDNG}_{a,f,r} \overline{\text{PRICE}}_f \\
 & \text{s.t.} \sum_f \text{FEDNG}_{a,f,r} \overline{\text{NUTR}}_{f,c} \geq \overline{\text{LEVL}}_{a,r} \overline{\text{REQU}}_{a,c,r} \quad \forall a, c
 \end{aligned} \tag{1}$$

¹ Compare e.g. HAZELL & NORTON (1986), 263 ff., KASNAKOGLU & BAUER (1989)

where $FEDNG_{a,f,r}$ is the quantity of the feedingstuff f fed to animal category a in region r to be determined and "bars" over variables denote known values. The constraints state that the requirements of the animals must be covered by an appropriate feed mix. The objective minimises total feed cost. We should notice that as long as all feedingstuff can be bought in unlimited quantities, the animal categories can be solved independently from each other without affecting the solution of (1). A minimum requirement of (1) from the view point of model calibration is feasibility in the base year. That can be achieved by an appropriate adjustment of the coefficients NUTR and REQU. It will be shown below how Maximum Entropy econometrics can be put to work to solve that problem.

However, not only prices and herd sizes are known in the base year, but the quantities fed at national level as well, at least for tradable feedingstuffs. There is little hope that solving (1) for all regions and adding up the resulting feed use will reproduce the observed feed quantities at national level without prior calibration. Already a "slight" deviation of 5 % in sensible markets such as cereals would surely irritate interested policy makers. Table 1 shows the deviation in Germany obtained from (1) if "hypothetical" requirements are used². The model is not allowed to use more of any feed category nationally than observed. Gras, silage and roots are assumed to be not tradable, and consequently their feeding quantities are fixed at regional availability. Since milk and sugar beet quotas fix sales, the feed quantities of milk and raw milk are fixed as the residual between sales at quota level and production (here also exogenous).

Table 1: Feed use from non calibrated model, Germany

Feed	Quantity used	% of base year
DHAY - hay	2421.685	100.00
STRA - straw	1784.057	100.00
FCER- cereals	17942.52	96.74
FPRO - protein rich	3818.568	38.94
FENE - energy rich	8713.235	100.00
FMIL - milk based	189.46	100.00
FOTH - other	0.0	0.00

When interpreting the results, one should keep in mind that the appealing looking 100 % values for all non-tradables (raw milk, root crops, silage, graze & grazing) are due to fixed values and results for rich energy and milk based feed are based on (obviously binding) upper bounds. Nonetheless, the program "squeezes" 9 Mio. t of cereals, protein rich, milk based and other fodder out. The results shall be sufficient to prove that a calibration is needed.

2.2 Second round: let the LP hit the target ...

Introducing an adding up constraint for total use of the different feedingstuffs at the national level (FEDUSEN) and fixing it's value to base year levels leads to:

² An exact definition of "hypothetical" will be given later. A slight increase in max. fiber detergent was necessary for cows to achieve feasibility. The feed aggregates are defined according to the SPEL-EU data base (WOLF 1995).

$$\begin{aligned}
& \min \sum_{a,f,r} \text{FEDNG}_{a,f,r} \overline{\text{PRICE}}_f \\
& \text{s.t.} \sum_f \text{FEDNG}_{a,f,r} \overline{\text{NUTR}}_{f,c} \geq \overline{\text{LEVL}}_{a,r} \overline{\text{REQU}}_{a,c,r} \quad \forall a, c, r \\
& \sum_{a,r} \text{FEDNG}_{a,f,r} = \overline{\text{FEDUSEN}}_f \quad \forall f
\end{aligned} \tag{2}$$

The last line ensures that the known quantities fed at national level are met. Unfortunately, problem (2) is not longer a cost minimisation problem, because quantities to be fed at national level and their prices are known! Consequently, the value of the objective function in (2),

$$\sum_{a,f,r} \text{FEDNG}_{a,f,r} \overline{\text{PRICE}}_f = \sum_f \overline{\text{FEDUSEN}}_f \overline{\text{PRICE}}_f \tag{3}$$

is a given constant. If we plug (2) in a solver, it cannot squeeze out any quantities since total use is fixed at national level. It will simply distribute the feed over regions and animals so that the requirements of animals are met. When the first feasible solution for the requirement constraints is found, it will stop. The distribution will be arbitrary, with potentially devastating consequences for the feed costs of the regional activities. Naturally, some of the constraints in (1) will be binding, but which ones will be arbitrary as well (as will be the dual values attributed to them).

2.3 Shooting sharp, but somewhat short: naive PMPs for feed

Readers familiar with the on-going discussion on Positive Mathematical Programming (PMP) will now tend to lay back and relax because they know already a nice solution to the calibration problem. Their obvious idea will be to use the dual values on the variables FEDUSEN to define additional non-linear terms to be added to the objective function³. Naturally, perfect calibration will be guaranteed by the PMP methodology. But what about the simulation behaviour?

If the model without calibration bounds produces a solution dramatically different from the observed one (see table 1), the influence of the PMP terms for feed on the simulation behaviour will be tremendous. Consequently, the effect of the requirement constraints will be small, hard to judge and depend on the quite arbitrary solution from the calibration step. The goal to describe the substitution possibilities in feeding by an appropriate set of constraints would surely be missed. Perhaps the best advice with such a solution would be to leave them out completely.

Instead, appropriate own and cross-cost term between feeding activities would need to be introduced to describe the cost minimising behaviour in feeding. The technological substitution possibilities between feedingstuffs which we aimed at describing by the constraints in (1) would be mostly hidden as dual information in these cost terms. Furthermore, the arbitrary solution from (1) would define the marginal costs of the animal activities entering the estimation of PMP terms for calibration of levels.

But there are further problems related to a PMP approach. Since the most expensive feedingstuff will always be squeezed out first, dual values of the corresponding calibration constraints will be equal to exogenous prices. Cross-regional or time series analysis of the duals from a sample of regional models will hence not produce any additional information.

³ PMP for feed are used, for example, in the RAUMIS model (CYPRIS 1999).

Without observed variance, a reliable estimation is not possible. The - at first sight - appealing PMP solution will consequently not work well, if no further information is available.

The essence of the argumentation above is that the use of an aggregated cost minimisation model in simulation runs is only sensible, if the constraints based on the coefficients (NUTR and REQU) can sufficiently explain observed feed use in the base year. Therefore, the "hypothetical" requirements must first be calibrated with respect to observed feeding quantities in the base year. Then, as a last resort, PMP can be put to work.

3 "Required requirements" or pushing the LP to the border

The general problem is then to define a set of requirements which calibrates the feed cost LP as close as possible to statistically observed quantities. It is solved by the following steps.

(STEP 1.) **Definition of nutritional content** (NUTR) of feedingstuffs. Whereas for cereals and other traded feedingstuffs, the information about their content of energy, protein etc. is quite accurate, doubt may be raised concerning fodder (gras, grazings, hay, silage etc.), both related to yield estimates and nutritional content. However, in order to not further complicate the framework, nutrient contents are treated as given constants. Hence, any error in the nutrient content is mapped into the correction of the nutritional requirements of the animal activities in step 3.

(STEP 2.) **Definition of nutritional requirements** (REQU) for each animal category in each region based on so-called requirement functions (NASUELLI ET. AL. 1997) for energy, crude protein, min. dry matter. For certain animal categories, additional requirements or constraints are introduced (lysine, max. dry matter intake, neutral detergent fibre etc.). For some of the animal categories they reflect differences in regional yields per head, for example the milk yield per cow. The underlying functions are based on a literature search and can be understood as the technological frontier under a controlled experimental environment.

(STEP 3.) **Calibration of these hypothetical requirements and constraints** so that they are as far as possible binding for the observed feed quantities. The necessity of calibrating the theoretical requirements can be easily understood when taking into account the control cost on farm level related to work exactly on the technological frontier: the nutrient content of feedingstuffs must be carefully checked and the intake of each feed per animal exactly weighted. Otherwise, one risks to starve the animal, to damage their health and to reduce yields, with high costs involved. To exclude that risk, farmers will feed securely more than theoretically required.

3.1 Three star dinner for calibration fans: Constrained entropy maximisation

The calibration process is based on the information comprised in the set of "hypothetical" requirements and feeding constraints (energy, crude protein, dry matter; max. dry matter intake) and the known quantities fed in the base year. As the later ones are "hard" data to meet, they serve as constraints for our calibration model. The Maximum Entropy criterion will minimise deviation from appropriate starting points based on the "hypothetical" requirements and will take the effect on the costs of the individual production activities into account.

In contrast to most other econometric techniques, the Maximum Entropy approach allows the estimation of parameters in the case of ill-posed problem. i.e. if the number of observations is less than the number of parameters (GOLAN, JUDGE & MILLER 1996). The parameter estimates

are probability weighted linear combinations of given support points (SUP). The objective prefers the “flattest”, i.e. least informative distribution of the probabilities (PROB).

In our case, the expected value for each parameter $E[p]$ is a linear combination of $k = 4$ support points SUP_k ⁴ weighted with probabilities $PROB_k$:

$$E[p] = \sum_k PROB_k \cdot SUP_k \quad (4)$$

with

$$\sum_k PROB_k = 1 \quad (5)$$

Since the model covers 13 animal categories, up to 6 requirements restrictions per animal category and up to 46 regions per Member State, the number of parameter exceeds 3000 to be estimated for some Member State⁵.

At first glance, one may wonder why the parameters for all regions are estimated simultaneously. However, the constraints of the problems are the given national, and - in some cases - regional quantities fed in the base year. As changes in the requirements in one region are likely to influence aggregated use at national level as well, a simultaneous solution is necessary. However, to ease the computational burden, the problem is solved in two steps:

1. Correction factors per animal and requirement/constraint are estimated for the national averages, the later ones based on regional herd sizes as weights.
2. The correction factors at national level are then used as the basis to define regional supports.

3.2 Antipasti or first task: Definition of support points

Requirements

Generally, support points for ME problems are based on a-priori information. The higher the spread of the supports, the weaker their influence on the final solution. As the flattest distribution is reached when all probabilities are equal, supports should be centred around a plausible expected value for the parameter to be estimated. In our case, the "hypothetical" requirements are unfortunately not such plausible expectations. As explained above, they represent technological frontier not applicable to the sectoral average.

⁴ The variance of the ME-estimates approaches a lower limit if the number of support points goes to infinity (GOLAN, JUDGE, MILLER 1986, p.139). In tests with a simple "two parameter" - "one constraint" ME problem, the effect of increasing the number of support points was judged significant only up to four support points. Beyond four support points, parameter estimates still kept changing systematically, but the change was close to the computational accuracy of the computer used. Since the computational burden of the solver grows dramatically with increasing number of support points, four supports were chosen for the analysis.

⁵ Tests where the requirements for an animal category in a region were estimated as the product of *general* correction factors - one for each region, animal category and requirement/constraint - had to be abandoned. The result contradicted the expectation of the author that the reduction from 3000 to 13 + 6 + 46 parameters would ease the solution of the problem. It turned out that (1) the small number of parameters did not allow for enough flexibility as curious data constellations for some regions and animal categories forced corner solutions if not infeasibilities and (2) that the definition of the expected value for the final correction factor per animal, region and requirements as a product of three parameters lead to non-linear constraints hard to crack by the solver. Further tests with intermediate solutions, for example individual correction factors per animal category and requirements multiplied with regional correction factors were abandoned due to similar problems.

If problem (1) - based on the "hypothetical" requirements - can squeeze out large quantities and hence underestimates the feeding costs, it is a clear hint that these requirements are either too low, not complete or the nutrient content of the feedingstuffs is too high. Anyway, as the two last problems cannot be healed easily, all errors introduced by aggregation problems, missing or incorrect information must be covered by adjusting our few constraints. The question is not if specific requirements in our problem are "correct" in the sense that a farmer or the feed compound industry would use it in determining the optimal mix, but if they are suitable in describing the aggregated substitution possibilities for feedingstuff.

Given the large number of feedingstuff, their differences in content in nutrients and prices, we would expect the sector working more or less exactly on the aggregated frontier of the "real" substitution set, i.e. that most of the constraints of problem (1) would be binding. We must hence look out for a suitable centre point for our supports for the individual requirements for which we would expect that the constraints are binding. Nevertheless, the supports should still relate to our "hypothetical" requirements. As the first step, we calculate base year relations between total "hypothetical" requirements $REQU^h$ and total deliveries for each constraint c in (1) at the national level:

$$REL_c = \frac{\sum_{a,r} LEVL_{a,r} REQU_{a,c,r}^h}{\sum_f FEDUSE_f NUTR_f} \quad (6)$$

With just one sectoral constraint, say energy requirements, the correction factor expressed by REL could be applied too all requirement coefficients directly and would ensure that the energy constraint is "just" binding. No ME estimation would be needed. However, for a complex layout with up to 6 requirements, fixed availability of certain feedingstuff in certain regions, applying the factors to all requirements would lead to infeasibilities.

The relations from (6) together with hypothetical requirements at national level (ms, based on national average yield) are used to define the support points for the calibration step at national level:

$$SUP_{a,c,ms} = \{0.1, 0.7, 1.3, 1.8\} REL_c REQU_{a,c,ms}^h \quad \forall a, c \quad (7)$$

In the second step, regional specific support points are based on the point estimates obtained at national level $E[REQU_{ms}]$:

$$SUP_{a,c,r} = \{0.1, 0.7, 1.3, 1.9\} REQU_{a,c,r} \frac{E[REQU_{a,c,ms}]}{REQU_{a,c,ms}^h} \quad \forall a, c \quad (8)$$

Feed costs

As feeding accounts for a large part of costs in animal production, correction of requirements should take the effect on costs into account. Therefore, support points for feeding costs (COST) are defined based on the feeding costs and revenues reported in the SPEL-EU data base. Expectations are centred around the maximum of feeding costs, 80 % of total costs and 30 % of total revenue reported in the data base:

$$SUP_{a,cost,r} = \{0.01, 0.7, 1.3, 1.9\} \max(0.3 REVENUES_{a,ms}, 0.8 TOIN, FEDCST_{a,ms}) \quad \forall a, c \quad (9)$$

3.3 The main course: The ME problem at work

Expected values for requirements and feeding costs are defined by the endogenously determined probabilities and the support points defined above:

$$\begin{aligned} E[\text{REQU}_{a,c,r}] &= \sum_k \text{PROB}_{a,c,r,k} \cdot \text{SUP}_{a,c,r,k} \quad \forall a, c, r \\ E[\text{COST}_{a,r}] &= \sum_k \text{PROB}_{a,\text{cost},r,k} \cdot \text{SUP}_{a,\text{cost},r,k} \quad \forall a, r \end{aligned} \quad (10)$$

Probabilities observe the adding up conditions:

$$\begin{aligned} \sum_k \text{PROB}_{a,c,r,k} &= 1 \quad \forall a, c, r \\ \sum_k \text{PROB}_{a,\text{cost},r,k} &= 1 \quad \forall a, r \end{aligned} \quad (11)$$

Following the notation established in part 1, the requirements defined per day and head are covered by:

$$E[\text{REQU}_{a,c,r}] \text{LEVL}_{a,r} \text{DAYS}_a / 1000. \leq \sum_f \text{FEDNG}_{a,f,r} \text{NUTR}_f \quad \forall a, c, r \quad (12)$$

Adding up the fed quantities over the herd sizes, the total feed use per region is defined as

$$\text{FEDUSE}_{f,r} = \sum_a \text{FEDNG}_{a,f,r} = \quad \forall f, r \quad (13)$$

Since some of the feedingstuffs (silage, gras, other root crops) are assumed to be not tradable between regions, the above equation is treated as an equality constraint where FEDUSE is fixed to the observed production quantities.

For all other feedingstuffs, the following adding up constraints at the national level define binding equality constraints for the calibration process where FEDUSEN defines the exogenously fixed, observed quantities fed at Member State level:

$$\sum_r \text{FEDUSE}_{r,f} = \text{FEDUSEN}_f \quad \forall f \quad (14)$$

Finally, the objective function with the Entropy criterion is defined as

$$\begin{aligned} H(\text{PROB}) &= - \sum_{a,c,r,k} \text{PROB}_{a,c,r,k} \log(\text{PROB}_{a,c,r,k}) \\ &\quad - \sum_{a,r,k} \text{PROB}_{a,\text{cost},r,k} \log(\text{PROB}_{a,\text{cost},r,k}) \end{aligned} \quad (15)$$

4 Avoiding collateral damage: modelling regional trade in hay and straw

According to the SPEL/EU data base, hay and straw are thought to be not tradable across national borders (WOLF 1995, p. 140). To ensure zero-net trade at national level in simulation runs, two solutions are possible:

1. Linking all regional model in a Member State with endogenous price determination

2. Solving the regional models in a loop and designing an external for price determination leading to zero net-trade

The first solution is the easy one if no computational restrictions apply. However, in the case of CAPRI up to 46 regions have to be linked simultaneously. Additionally, the simultaneous solution per Member State would not be sufficient, because endogenous prices for all internationally traded products must still be determined by interaction with the market module. A simultaneous solution of all regional supply models and the market model is not feasible. But if a loop around the supply and market part is necessary anyway, it seems appropriate to embed price determination for hay and straw in it as well.

The price determination is based on a Taylor-approximation⁶:

$$f(x + \Delta x) \approx f(x) + f'(x)\Delta x \quad (16)$$

where we describe the net trade at Member State level (NT) as a function of an uniform clearing price (PRICE). In each iteration i , we estimate a price change $\Delta \text{PRICE}^{i+1} = \text{PRICE}^{i+1} - \text{PRICE}^i$, i.e. - for readers familiar with solvers - a step length which is expected to lead to zero net trade. As the first derivative of net trade with respect to the price is unknown, $\Delta \text{PRICE}^{i+1}$ is estimated based on the change in the last iteration:

$$\begin{aligned} 0 &\equiv \sum_r \text{NT}_r^{i+1} \equiv \text{NT}^{i+1}(\text{PRICE}^i + \Delta \text{PRICE}^{i+1}) \\ &\approx \text{NT}^i(\text{PRICE}^i) + \frac{\Delta \text{NT}^i}{\Delta \text{PRICE}^i} \Delta \text{PRICE}^{i+1} \\ \Rightarrow \Delta \text{PRICE}^{i+1} &= -\text{NT}^i \frac{\Delta \text{PRICE}^i}{\Delta \text{NT}^i} \end{aligned} \quad (17)$$

Note, that this approach would directly yield zero net trade in iteration $i+1$, if the relation $\Delta \text{NT}/\Delta \text{PRICE}$ is a constant, i.e. independent of ΔPRICE (which is a rather unlikely case for the underlying programming models). For the first step, or if the algorithm stalls, we use the following "best guess":

$$\text{PRICE}^{i+1} = \text{PRICE}^i \frac{\text{PROP}^i - \text{NT}^i}{\text{PROP}^i} \quad (18)$$

Furthermore, it seems not plausible to model trade in regionally produced feedingstuffs without taking into account transport and transaction costs. Therefore, the uniform price described above is regionalised depending on the trade quantity's share of production⁷. The more is sold, the smaller the price received and vice versa. It is assumed that PRICE_r in an iteration drops by 50 % of the base year price $\text{PRICE}_{b,r}$ if a quantity equivalent to total base year production $\text{PROP}_{b,r}$ is marketed:

$$\text{PRICE}_r = \text{PRICE}_b - \frac{1}{2} \text{PRICE}_{b,r} \frac{\text{NT}_r}{\text{PROP}_{b,r}} \quad (19)$$

⁶ A similar mechanism is used in the FAO's World Food Model to find market clearing prices (see e.g. FROHBERG & BRITZ 1995)

⁷ Price dependencies for regionally traded feedingstuffs were incorporated in the German Sector Model RAUMIS as well (SCHMITZ 1994)

Equation (19) is incorporated in the objective function of the regional models and net trade as well as the regional prices are endogenous variables.

5 Salvation for linear programmers: derivation of PMP terms for feed

The requirements based on the solution of the entropy problem described in part 2 as well as the trade price determination described in part 3 are then integrated in a framework where all the regional supply models for a Member State are linked together. This simultaneous solution is used only once for the calibration step whereas regional model will be solved independently in simulation runs.

Calibration constraints ensure

- that the activity levels of the base period are met
- that observed national feed quantities are used up
- that observed regional production quantities of graze and silage are fed.

The "normal" constraints of the framework which are identical to the one applied in simulation runs consist of the requirement constraints, area restrictions and political restrictions such as selling quotas for milk and sugar and the CAP set-aside regime.

In the first calibration step, optimal regional trade with hay and straw is derived. Regional prices differ from the uniform national price according to (19).

In the next step, the regional models are solved independently with regional feed use fixed at the results observed in the first step. On the one hand, the dual values of the calibration constraints are used in the Maximum Entropy estimation of the quadratic cost function for the activities (HECKELEI & BRITZ 1999). On the other hand, dual values for the fixed feed quantities are obtained.

As in the case of production activities, the marginal values for feed are mapped into non-linear terms of the objective⁸. In order to get a simple and easy to interpret definition of these non-linear terms for feed use, the quadratic terms BF are based on an own-price point elasticity of -0.5 for regional feed use. Linear terms AF ensure that the dual values obtained are met and that the model calibrates perfectly to the base year:

$$BF_{f,r} = -0.5 \frac{PRIC_{f,b}}{FEDUSE_{f,b,r}} \quad (20)$$

$$AF_{f,r} = -\lambda_{feduse(f,b,r)} - BF_{f,r} FEDUSE_{f,b,r}$$

Note that "PRIC" in (20) is the uniform national price as found in the SPEL/EU data base and differ hence from the price for the regionally traded feedingstuffs hay and straw obtained by the mechanism described in part 3.

⁸ Positive Mathematical Programming in the context of feed distribution is also used in the German sector model RAUMIS, since 1997 (CYPRIS 1999). However, non-linear cost terms are introduced for *each* feeding activity (for example "wheat fed to pigs") and not for total regional feed use as done here.

6 Summary

When judging the solutions discussed above, the overall model's objectives and structure as well as the data availability must be kept in mind. Little is known statistically about feeding practises in different regions across Europe. Besides, the main analytical objective of the CAPRI modelling system is directed towards policy impacts on regional and aggregate activity substitution, i.e. on crop levels and herd sizes, and on the resulting impacts on and feed-backs from the markets.

The distribution of individual feedingstuff to individual animal categories is of minor importance in the overall context and matters mostly from the view point of influencing the overall allocation behaviour.

The supply module deals with bulks of traded feeding stuff only: cereals, rich protein, rich energy and other feed. Silage, graze, root crops and raw milk are modelled on a single product basis. Without further remedies, the few linear requirement constraints would result in a quite jumpy behaviour of the regional model. A successful integration with the market module would be impossible. Therefore, prices for regionally traded feed depend on the net trade and PMP-terms are introduced for total regional feed use for each feeding stuff in the supply module.

The quantities fed in the supply module are exogenous fixed variables in the market module which determines the mix of the bulks, for example the share of wheat, barley and maize in the cereals aggregate simultaneously with the prices for the components (BRITZ 1998). The elasticities used in double log function determining the shares are partly based on estimated cost functions for the German feed compound industry (BRITZ & SIEBER 1998). The resulting new price for the bulks as well as their new nutrient content are then handed back in the next overall model iteration to the supply module.

The overall structure is hence a mixture of primal and dual approaches. Hopefully, the solution mainly integrates the advantages of the different solutions instead of combining all possible draw-backs.

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