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A Maximum Entropy Approach to the Calibration of a Highly Differentiated Demand System

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Abstract

In this paper a Maximum Entropy approach is presented to derive a set of demand elasticities parameters for a differentiated system demand functions. The calibration procedure ensures the system to be consistent with micro-economic theory (including the curvature conditions) at a base period. At the same time restrictions stemming from plausibility considerations are incorporated. The procedure is applied to EU member states using the GAMS programming tool.

1 Introduction

If it is to be relevant for policy making, quantitative agricultural sector modelling has to be highly differentiated. This led to the specification of a detailed commodity list in the case of the SPEL/EU-MFSS model (Weber 1995) and of the CAPRI project (Britz & Heckelei 1997). A complete estimation of the demand systems at this level of differentiation (for each EU country) based on a relatively short time series of yearly data would be a formidable task, at least.

Since the days of SWOPSIM, most modelling groups using large scale multi-market models evaded the task to estimate the parameters for their demand systems. Instead the practice has become common to borrow these parameters from other studies or from parameter sets collected by other modelling groups, a procedure labelled "heuristic" by friendly observers.

Unfortunately, and the same is true in our case, the exact sources and treatments of these borrowed parameters are often unknown. Definitions and data underlying the original estimation do not match those of the models where they happen to be used finally. To some extent therefore, the common practice to collect the parameters of a model from different sources is rather haphazard.

In this situation microeconomic theory offers at least some guidance to the appropriate adjustment of borrowed parameters. Without such adjustment, borrowed parameters would involve in most cases significant violations of fundamental properties of micro-economic consumer theory when transferred into the large scale model. The reasons for these violations are as just mentioned, differences in product definitions, in data sources, and the mixing of parameters from different sources. Often, the borrowed parameter sets have never been consistent with micro-economic theory in the first place.

In addition to microeconomic theory, a priori expectations as to reasonable magnitudes and relationships might help in the task to extract a reliable set of demand parameters from the available evidence in the literature. One such expectation is net substitutability between all goods. Income effects aside, we would expect that an increase in the price of, say, beef would *increase* demand for other meats to some extent (see e.g. Heckelei, Mittelhammer, Wahl 1997) and, similarly, to have a nondecreasing but potentially very small effect on less closely related goods.

2 Methodology of calibration

2.1 Constrained entropy maximisation

The calibration process is based on a set of given, uncalibrated elasticities (see 3.2). The calibration process minimises deviations from these starting points based on Maximum Entropy as the objective and constraints derived from consumer theory.

In contrast to most other econometric techniques, the Maximum Entropy approach allows estimation of parameters in the case of underdetermined systems. The parameters are probability weighted linear combinations of given support points and the objective prefers the “flattest”, i.e. least informative, distribution for the probability weights. In our case, each compensated price elasticity[1] $\hat{e}_{ij}$ and income elasticity e_{iy} is a linear combination of k=2 support points sup_{ijk} centred around the uncalibrated starting points, i.e. around the elasticities selected from the literature. Each support point is weighted with probability pr_{ijk}, e.g.:

$$\hat{e}_{ij} = \sum_k pr_{ijk} \cdot sup_{ijk} \quad \forall i, j \tag{1}$$

where probabilities add up:

$$\sum_k pr_{ijk} = 1 \quad \forall\, i, j \tag{2}$$

Entropy[2] is at its maximum when each support point carries the same probability $pr_{ijk} = 0.5$:

$$E = \sum_i \sum_j \sum_k pr_{ijk} \log(pr_{ijk}) + \sum_i \sum_k pr_{iyk} \log(pr_{iyk}) \tag{3}$$

However, the attractive solution $pr_{ijk} = 0.5$ is usually inconsistent with the constraints imposed. The constraint on the income elasticities is adding up:

$$\sum_i e_{iy} s_i = 1 \quad \forall\, i \tag{4}$$

where

e_{iy} = income elasticity of product i

s_i = $p_i \cdot q_i / y$ = share of product i

p_i = consumer price of product i

q_i = consumed quantity of product i

y = total expenditure

Theoretical constraints for price elasticities are more easily expressed in term of Hicksian elasticities. Uncompensated (Marshallian) elasticities e_{ij} collected from different sources are therefore converted to compensated (Hicksian) elasticities $\hat{e}_{ij}$ using the Slutzky equation:

$$\hat{e}_{ij} = e_{ij} + s_j e_{iy} \tag{5}$$

---

1 Note that elasticities relating to compensated (Hicksian) demands are marked with a caret (ê) in this paper while uncompensated (Marshallian) elasticities are denoted without the caret (e)

2 In the actual implementation in the program, we added a weight of 0.5 to the cross price elasticities (i ≠ j) to account for the symmetry condition.

where

$\hat{e}_{ij}$ = compensated price elasticity of product i wrt. price j

e_{ij} = uncompensated price elasticity of product i wrt. price j

Symmetry of Hicksian substitution terms may be written in terms of compensated elasticities as follows:

$$\hat{e}_{ij}\frac{q_i}{p_j}\overset{!}{=}\hat{e}_{ji}\frac{q_j}{p_i} \Leftrightarrow \hat{e}_{ij}s_i\overset{!}{=}\hat{e}_{ji}s_j \ , \ \forall i,j \tag{6}$$

Homogeneity implies that compensated demands do not change if all prices change by the same percentage[3]:

$$\sum_j \hat{e}_{ij}\overset{!}{=}0, \ \forall i \tag{7}$$

The most difficult practical problem is the imposition of appropriate constraints to ensure the right curvature of the demand system, i.e. negative semidefiniteness of the matrix of Hicksian substitution terms. A sufficient condition would be that all minors that may be formed along the main diagonal alternate in sign (or are 0), but this is not practical beyond the third order. Checking the minors up to order three provides only a necessary condition, therefore. The conditions applying on the matrix of substitution terms in the first place may be shown to carry over to compensated elasticities. Instead of using inequalities conditions greater/lower *zero*, it turned out useful from a computational point of view to strengthen the inequalities slightly in the following way:

$$\begin{aligned} &\hat{e}_{ii} < -1.E-3, \quad \forall i \\ &\hat{e}_{ii}\hat{e}_{jj} - \hat{e}_{ij}\hat{e}_{ji} > 1.E-6, \quad \forall i,j \\ &\hat{e}_{ii}\hat{e}_{jj}\hat{e}_{kk} + \hat{e}_{ij}\hat{e}_{jk}\hat{e}_{ki} + \hat{e}_{ik}\hat{e}_{ji}\hat{e}_{kj} - \hat{e}_{ik}\hat{e}_{jj}\hat{e}_{ki} - \hat{e}_{ij}\hat{e}_{ji}\hat{e}_{kk} - \hat{e}_{ii}\hat{e}_{jk}\hat{e}_{kj} < -1.E-9, \ \forall i,j,k \end{aligned} \tag{8}$$

Because this still does not guarantee the right curvature (higher order minors are not checked), the calibrated set is subsequently tested and potentially corrected using the so-called modified Choletzky decomposition. In contrast to the standard Choletzky-decomposition which provides a well-known sufficient test for semidefiniteness, the modified Choletzky decomposition applies “optimal” correction factors to the diagonal elements of the matrix which just ensure the right curvature (Gill et. al. 1981, pp. 110)[4]. Because sufficiency of curvature is checked and imposed only after the NLP-calibration problem, a loop had to be used which will be repeated until the NLP-calibration procedure produced a solution which is negative semidefinite without further correction. Our formulation has proven practical in the sense that the results from the maximisation subject to constraint (8) on the first minors are in most cases already negative semidefinite.

The final set of constraints imposes net substitutability:

$$\hat{e}_{ij} > 0, \ \forall i \neq j \tag{9}$$

---

[3] Homogeneity of uncompensated demands follows from this and adding up.

[4] Usually, the algorithm is used in conjunction with modified Newton Methods in solvers and embedded, for example, in MINOS.

The last condition turned out crucial to ensure plausibility of results. It also helped in the optimisation procedure for which GAMS is used in connection with CONOPT2 as the non-linear solver. Some fine tuning of tolerances was necessary, however, to achieve acceptable solution times.

2.2 Hierarchical initialisation

Estimation of large scale demand systems is usually done in hierarchical systems, because this helps to economise on parameters. Therefore published evidence usually either refers to rather aggregated elasticities, say of "meat", as in Michalek, Keyzer 1990 or to smaller systems, say of beef, pork etc. with group expenditure given (e.g. Heckelei 1997, Rickertson 1996). Consequently there might be some a priori information on the elasticities of beef with respect to the price of pork but hardly any directly estimated evidence on the elasticity of beef with respect to butter.

Therefore the calibration procedure used here also starts by calibrating the substitution effects and expenditure elasticities inside of the groups (cereals, meat, fruits & vegetables, milk products & eggs), followed by a calibration of inter-group effects and expenditure elasticities for the groups.

The details of this procedure build on two stage budgeting conditions. One of the more general cases permitting two stage budgeting starts from additivity of the direct utility function (Deaton, Muellbauer1980, p 131)

$$v(p,y) = \max_{q}\{u(q_1) + ... + u(q_N) : p'q \le y\} \tag{10}$$

where

$q_G = (q_{G1} \ldots q_{Gi} \ldots q_{Gn_G})'$ = vector of consumption quantities in group G

$q = (q_1 \ldots q_G \ldots q_N)'$ = combined vector of consumption quantities

The indirect utility function of this problem should assume the Gorman polar form:

$$v(p,y) = \max_{Q_G}\left\{\sum_G (f_G(Q_G) + a_G(p_G)) : \sum_G P_G(p_G) Q_G \le y\right\} \tag{11}$$

where group aggregate quantities Q_G represent an index defined from dividing group expenditures y_G by group price indices $P_G(p_G)$:

$$Q_G \equiv y_G / P_G(p_G) \tag{12}$$

and group indirect utility $v_G(p_G, y_G)$ and expenditure functions $e_G(p_G, u_G)$ relate as follows to disaggregated values:

$$\begin{aligned} v_G(p_G, y_G) \equiv \max_{q_G}\{u_G(q_G) : p_G' q_G \le y_G\} &= f_G(y_G / P_G(p_G)) + a_G(p_G) \\ \Leftrightarrow u_G - a_G(p_G) &= f_G(y_G / P_G(p_G)) \\ \Leftrightarrow y_G &= P_G(p_G) f_G^{-1}(u_G - a_G(p_G)) \equiv e_G(p_G, u_G) \\ &= P_G(p_G)\, Q_G \end{aligned} \tag{13}$$

These assumptions imply the following system of demand functions conditioned on group expenditures:

$$
\begin{aligned}
q_{11} &= d_{11}(p_{11},\ldots,p_{1n_1},y_1)\\
&\vdots\\
q_{1n_1} &= d_{1n_1}(p_{11},\ldots,p_{1n_1},y_1)\\
&\vdots\\
q_{N1} &= d_{N1}(p_{N1},\ldots,p_{Nn_N},y_N)\\
&\vdots\\
q_{Nn_N} &= d_{Nn_N}(p_{N1},\ldots,p_{Nn_N},y_N)
\end{aligned}
\tag{14}
$$

where group expenditures are determined in an upper level demand system:

$$
\begin{aligned}
y_1 &= P_1 Q_1 = P_1 D_1(P_1,\ldots,P_N,y)\\
&\vdots\\
y_N &= P_N Q_N = P_N D_N(P_1,\ldots,P_N,y)
\end{aligned}
\tag{15}
$$

In this hierarchical system, the overall or "integrated" (Marshallian) elasticities ε_{GiHj} may be derived from the lower level elasticities e_{Gih} and e_{Giy} and the top level elasticities E_{FG} and E_{Gy} as follows, first for the income elasticities:

$$
\begin{aligned}
\varepsilon_{Giy} &= \frac{\partial q_{Gi}}{\partial y}\frac{y}{q_{Gi}} = \frac{\partial q_{Gi}}{\partial y_G}\frac{y_G}{q_{Gi}}\frac{\partial y_G}{\partial y}\frac{y}{y_G}\\
&= \frac{\partial q_{Gi}}{\partial y_G}\frac{y_G}{q_{Gi}} P_G \frac{\partial Q_G}{\partial y}\frac{y}{Q_G}\frac{1}{P_G} = e_{Giy}E_{Gy}
\end{aligned}
\tag{16}
$$

where

ε_{Giy} = elasticity of good i ($\in$ group G) wrt. total expenditure y

e_{Giy} = elasticity of good i ($\in$ group G) wrt. group expenditure y_G

E_{Gy} = elasticity of group G consumption wrt. total expenditure y

For the price elasticities between items from different groups the following relationship (comp. de Haen, Murty, Tangermann 1982, eq. 22) may derived from (13) to (14) with some additional algebra:

$$
\varepsilon_{GiHj} = \frac{\partial q_{Gi}}{\partial p_{Hj}}\frac{p_{Hj}}{q_{Gi}} = \frac{\partial q_{Gi}}{\partial y_G}\frac{\partial (P_G Q_G)}{\partial P_H}\frac{\partial P_H}{\partial p_{Hj}}\frac{p_{Hj}}{q_{Gi}} = e_{Giy}E_{GH}s_{Hj}
\tag{17}
$$

where

ε_{GiHj} = uncompensated elasticity of good i ($\in$ group G) wrt. price j ($\in$ group H)

e_{Giy} = elasticity of good i ($\in$ group G) wrt. group expenditure y_G

E_{GH} = uncompensated elasticity of group G consumption wrt. group price H

s_{Hj} = $p_{Hj} \cdot q_{Hj} / y_H$ = share of good j ($\in$ group H) in group expenditure[5] y_H

5 For readers of the associated GAMS program we add the explanation that these within group shares follow from the shares in total expenditure as s_j/s_H .

The overall price elasticities within the same group turn out as follows (comp. de Haen, Murty, Tangermann 1982, eq. 21):

$$\varepsilon_{GiGh} = \frac{\partial q_{Gi}}{\partial p_{Gh}} \frac{p_{Gh}}{q_{Gi}} = \left. \frac{\partial q_{Gi}}{\partial p_{Gh}} \frac{p_{Gh}}{q_{Gi}} \right|_{y_G} + \frac{\partial q_{Gi}}{\partial y_G} \frac{\partial (P_G Q_G)}{\partial P_G} \frac{\partial P_G}{\partial p_{Gh}} \frac{p_{Gh}}{q_{Gi}} = e_{Gih} + e_{Giy} s_{Gh} (1 + E_{GG}) \qquad (18)$$

where

ε_{GiGh} = uncompensated elasticity of good i ($\in$ group G) wrt. price h ($\in$ group G)

e_{Gih} = uncompensated elasticity of good i ($\in$ group G) wrt. price h ($\in$ group G) with group expenditure y_G held constant

e_{Giy} = elasticity of good i ($\in$ group G) wrt. group expenditure y_G

E_{GG} = uncompensated elasticity of group G consumption wrt. group price G

s_{Gh} = $p_{Gh} \cdot q_{Gh} / y_G$ = share of good h ($\in$ group G) in group expenditure y_G

In practice, however, the combination of lower level elasticity systems, which are perfectly consistent within each group, with the top level system, which is perfectly consistent internally as well, does not automatically yield a consistent complete system. This is because the basic assumptions (11) to (13) for two stage budgeting imply stronger restrictions on elasticities than those from general demand theory. These stronger restrictions are very likely to be violated in lower and upper level systems which are calibrated using the Maximum Entropy approach presented in section 2.1 without further restrictions. Therefore the overall elasticities computed according to (16) to (18) are only initial values for a final calibration step which proceeds along the lines of section 2.1.

This final NLP problem with 40x40 elasticities per country and some hundreds of constraints would quickly exceed the capabilities of any solver without reasonable starting values. The sequential procedure starting with smaller lower level systems, an aggregate top level system, integration of preliminary results to starting values for the overall system and finally calibration of the complete system is likely to be indispensable for feasibility.

3 Data

3.1 The underlying data base of SPEL/EU-MFSS

Quantities

Data used in the calibration represent data for 1991-95 from the SPEL/EU-Data base. The data base covers about 50 agricultural raw and processed products. The set of raw products breaks down the product coverage of the Economic Accounts of Agriculture and can be named in that sense "complete". The list of processed products is relatively small (rice, molasses, potato starch, sugar, oils and cake/meals from rape, sunflower, soy, olives and other oils, milk powder, butter and other milk products).

Final demand quantities are equal to the position "Human consumption on market" (PCOM[6]). This refers to quantities stemming from national production and imports which are used for final human consumption. Eurostat claims that human consumption of processed products had

6 SPEL-EU code

been converted back over several processing stages (e.g. wheat <- wheat flour <- bread) in raw product quantities. Use of these quantities in the context of both SPEL/EU-MFSS and CAPRI has the important advantage that market balances are consistent ex-post with the supply domain of the SPEL/EU-Data base.

Prices

In contrast to demand quantities, consumer prices are not yet included in the SPEL/EU-Data base. However, consumer prices (CPRI) are calculated in the SPEL-MFSS model and in the CAPRI model, based on the work of Schein 1993, using the quantities from the SPEL/EU-Data base and statistical data from National Economic Accounts on consumer expenditures.

In most cases, data on aggregated consumer expenditure are available only on a higher product aggregation level than in the SPEL/EU-Data base. In order to derive prices for the individual components nonetheless, the following procedure is applied. First, the raw product value for an aggregate - human consumption multiplied with raw product prices - is subtracted from the consumer expenditure of the aggregate, say cereals. The residual is interpreted as processing costs etc. and distributed to the individual quantities. To this end it is divided by the added quantities of all components to obtain the average marketing cost per unit of raw product in the group. Consumer prices for the individual components of the aggregate, in our example soft wheat, durum wheat, rye, oats, maize etc. are hence equal to the raw product value plus the (uniform) marketing cost in the cereal group.

3.2 Sources of the starting values for demand elasticities

The starting values of the calibration have been based mostly on elasticities published in Schein 1993 or found in background files from Schein's work. These elasticities describe substitution inside and reaction to food expenditure changes of agricultural products which were taken to be organised in groups (cereals, meat, fruits and vegetables, butter and oils, sugar, milk products and eggs). Schein himself used de Haen, Murphy & Tangermann 1982, additional unknown sources and some calibration process.

The top level elasticities in Schein 1993 were borrowed from Michalek & Keyzer 1990 and this crucial source has been used in our calibration as well.

Due to the fact that several elasticities for single products (oats, rye, barley, table grapes, table olives, veal and vegetables oils besides rape oil) were missing in the published part, the following preliminary solution was applied based on a similar “parent” product where elasticities were at hand. The own price and expenditure elasticity was taken over and cross price elasticities inside of the group were set to 10 % of the existing “parent” own price elasticity.

Elasticities for Finland and Sweden where based on those for Denmark, in the case of Austria on German values and for Portugal on Spanish elasticities.

4 Results for an extended example

Instead of presenting the full set of final elasticities[7] in many tables we have decided to concentrate on a specific example, meat demand in Germany, which is a typical for our procedure.

[7] The full set of final elasticities is available upon request as a gams file.

The following table 1 presents the Marshallian input price elasticities as they have been preliminarily synthesised from the literature.

Table 1: Input values for Marshallian price elasticities of meat demand in Germany

	Beef	Veal	Pork	Sheep	Poultry	Expend.	Share
Beef	-0,41	-	-0,10	0,05	-	0,509	0,313
Veal	-	-	-	-	-	0,509	0,043
Pork	0,05	-	-0,39	-	-	0,472	0,518
Sheep	-	-	0,18	-0,41	-	0,509	0,010
Poultry	-0,10	-	-	0,12	-0,40	0,605	0,116

It is evident that a lot of gaps have to be filled, for example for veal. As explained above, the Marshallian elasticities will be derived from a parent product, in this case from beef demand. More precisely we take over the implied Hicksian price elasticity and the expenditure elasticity and put the (Marshallian) cross price elasticities of veal at 10% of the absolute value of the own price elasticity of the parent product beef. Expressed in terms of Hicksian elasticities we thus obtain the following set of elasticities:

Table 2: Input values for Hicksian price elasticities of meat demand in Germany

	Beef	Veal	Pork	Sheep	Poultry
Beef	-0,249	0,063	0,164	0,055	-
Veal	0,200	-0,249	0,264	0,005	0,059
Pork	0,198	-	-0,143	-	-
Sheep	-	-	0,439	-0,403	-
Poultry	0,089	-	-	0,127	-0,330

In any non-linear optimisation problem good starting values are helpful for solution speed and frequently they are indispensable to obtain a reasonable solution at all. Therefore the missing elasticities are initialised mostly with starting values following from selected microeconomic conditions (symmetry, homogeneity) or with reasonable default values (e.g. +0.001 for the Hicksian cross price elasticities).

In table 2, for example, all missing elasticities but those between poultry and pork may be derived from the symmetry conditions (6). Subsequently, the initialisation procedure checks whether homogeneity may be used to fill remaining gaps and a "second round" application of symmetry will yield additional starting values, in this case for poultry and pork. To speed up the solution, the expenditure elasticities taken from the literature (table 1) are also scaled to comply with adding up according to (4). The whole sequence of initialisation steps finally produces the starting values for the maximum entropy problem, in our example:

Table 3: Starting values of Hicksian price elasticities and expenditure elasticities for the maximum entropy problem in the meat demand subsystem in Germany

	Beef	Veal	Pork	Sheep	Poultry	Expend.	Share
Beef	-0,249	0,063	0,164	0,055	0,033	1,016	0,313
Veal	0,200	-0,249	0,264	0,005	0,059	1,016	0,043
Pork	0,198	0,022	-0,143	0,009	0,001	0,942	0,518
Sheep	1,695	0,022	0,439	-0,403	1,443	1,016	0,010
Poultry	0,089	0,022	0,091	0,127	-0,330	1,208	0,116

Our somewhat eclectic procedure above to obtain starting values is not meant to result in a fully consistent set of elasticities, because the conditions have not yet been imposed to hold *simultaneously*. In table 3, for example, the homogeneity condition (7) is evidently violated for veal and sheep. In addition, the high cross price elasticities for sheep contradict concavity. All elasticities whose starting values had to be derived in the initialisation procedure will carry a weight of zero in the objective function. Consequently, deviations from these technical starting values are not punished in the optimisation and the solver will pick those values compatible with the whole set of constraints and an optimal deviation of other elasticities from their given a priori information. The result of this optimisation for the meat sub system in Germany is the following:

Table 4: Solution values of Hicksian price elasticities and expenditure elasticities of the entropy maximisation in the meat demand subsystem in Germany

	Beef	Veal	Pork	Sheep	Poultry	Expend.
Beef	-0,237	0,023	0,179	0,000	0,035	1,016
Veal	0,167	-0,319	0,095	0,005	0,052	1,016
Pork	0,108	0,008	-0,163	0,006	0,040	0,942
Sheep	0,000	0,021	0,303	-0,457	0,133	1,016
Poultry	0,094	0,019	0,181	0,012	-0,306	1,208

A careful comparison will reveal that the deviation from the starting values tends to be much lower where some serious a priori information had been available (see table 1) rather than mere technical starting values (filled gaps in Table 3). These Hicksian elasticities correspond to the following set of Marshallian elasticities giving the reaction of demand *for given meat expenditure :*

Table 5: Implied Marshallian price elasticities after entropy maximisation for the meat demand subsystem in Germany

	Beef	Veal	Pork	Sheep	Poultry
Beef	-0,555	-0,021	-0,347	-0,010	-0,083
Veal	-0,151	-0,363	-0,431	-0,005	-0,066
Pork	-0,187	-0,033	-0,651	-0,003	-0,069
Sheep	-0,318	-0,023	-0,223	-0,467	0,015
Poultry	-0,284	-0,033	-0,445	0,000	-0,446

Evidently the income effect provides an additional push into the negative direction for all elasticities, leading in general to gross complementarity (with the exception of the poultry-sheep relationship) and to more elastic own price reactions. Compared to the input elasticities of table 1, the difference is crucially due to these income effects which have been increased in the calibration procedure to conform to the concept of group expenditure elasticities (compare tables 1 and 3).

However, even the elasticities of tables 5 are not yet the final results. What remains to be done is to integrate these within group elasticities e_{Gih} (including those for other groups such as cereals etc.) to an overall system using the relationships (16) to (18) given in section 2.2. Having done this, a final entropy maximisation produces the final Hicksian demand elasticities which may be converted to the following Marshallian demand elasticities ε_{GiGh} giving the reaction of meat demand to price changes with total expenditure (as opposed to meat expenditure) held constant:

Table 6: Implied Marshallian price elasticities for meat demand in Germany after final calibration of the overall system

	Beef	Veal	Pork	Sheep	Poultry	Expend.
Beef	-0,298	0,015	0,078	0,000	0,008	0,253
Veal	0,106	-0,327	0,001	0,003	0,029	0,253
Pork	0,047	0,000	-0,246	0,003	0,014	0,235
Sheep	-0,002	0,013	0,144	-0,457	0,110	0,253
Poultry	0,022	0,011	0,061	0,010	-0,333	0,301

These final price elasticities are again stronger positive than the elasticities given in table 5 because a positive term is usually added to the within group elasticities (see (18)). This turns gross complementarity again into substitutability for most relationships and renders the overall own price reaction less elastic. Income elasticities have been diminished as well compared to table 3 or 4, because group expenditure elasticities are multiplied with the elasticity of meat expenditure with respect to total expenditure (see (16)) which had been fixed to a value of 0.25 based on Michalek, Keyzer 1990.

As a final warning, it should be mentioned that the selected results from table 6 (and the full set of elasticities for EU member states available upon request) are not based on an extensive survey of the empirical evidence on demand elasticities estimated in the EU and elsewhere. This being said, it should be clear that the elasticities obtained so far have to be considered preliminary, compiled mainly to test the practical applicability of the approach suggested in terms of integrating a priori expectations in a theoretically consistent manner. Nonetheless, compared with older sets of elasticities, such as those used in Weber 1995, even our preliminary results should be an improvement. The next steps should be, of course, to do a systematic review of the available evidence and, as soon as time constraints permit, to estimate some of the elasticities using the SPEL data basis underlying the CAPRI system and other models as well.

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